Random data Cauchy theory for the incompressible Navier-Stokes equations

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Abstract. In this talk, we consider the random data Cauchy theory for the incompressible Navier–Stokes system. At first, we study the existence and uniqueness of the strong solution for the classical incompressible Navier–Stokes equations with the L^2 initial data and the periodic space domain $\mathbb{T}^3$. After a suitable randomization, we are able to construct the local unique strong solution for a large set of initial data in L^2. Furthermore, if $\|u_0\|_{L^2}$ is small, we show that the probability for the global existence and uniqueness of the solution is large. Then, we consider the generalized Navier-Stokes equations,

$$u_t + u \cdot \nabla u + (-\Delta)^\alpha u + \nabla \Pi = 0$$

where $x \in \mathbb{T}^N$ or $\mathbb{R}^N$, $N \geq 3$, and $\alpha \in (\frac{1}{2}, \frac{N+2}{4})$. After a suitable randomization, we obtain the existence and uniqueness of the local mild solution for a large set of the initial data in H^s, $s \in [-\alpha, 0]$, if $1 < \alpha < \frac{N+2}{4}$, $s \in (1 - 2\alpha, 0]$, if $\frac{1}{2} < \alpha \leq 1$. Specially, our result shows that, in some sense, the Cauchy problem of the classical incompressible Navier-Stokes equation is local well-posed for a large set of the initial data in H^{-1+}, exhibiting a gain of $\frac{N}{2}-$ derivatives with respect to the critical Hilbert space $H^{\frac{N}{2}-1}$. At last, we obtain the almost sure existence of global weak solutions with the initial data in $\mathbb{H}^s(\mathbb{T}^N), s \in (-1, 0)$. Specially, the global weak solution is unique when $N = 2$.(Based on the works with Daoyuan Fang and Lihuai Du)