Title: Newton's Method: Universality and Geometry
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Abstract: The lecture has 3 sections:
§**1**. Given functions $u, f : D \to D,\ D \subset \mathbb{R}$, if $u(x) = x - \frac{f(x)}{f'(x)}$ for all $x \in D$, we call f the *inverse Newton transform* of u, denoted $f = \mathbf{N}^{-1}u$. If $1/(x-u(x))$ is integrable, then

$$(\mathbf{N}^{-1}u)(x) = C \cdot \exp\left\{ \int \frac{dx}{x-u(x)} \right\},\ C \neq 0.$$

For such u, the iteration $x_+ := u(x)$ (away from its fixed points) is a Newton method on f, and the relations between (fixed points, monotonicity, of) u and (roots, convexity, of) f give a simple explanation of chaotic behavior, illustrated here for the logistic iteration, [1]
§**2**. A geometric interpretation of the complex Newton iteration, $z_+ := z - \frac{f(z)}{f'(z)}$, f analytic, [4], allows extending the results of §**1** to complex iterations. This is illustrated for the Mandelbrot set.
§**3**. An iterative method for minimizing a convex function $f : \mathbb{R}^n \to \mathbb{R}$ with an attained infimum, proceeds by bracketing the minimum value in nested, decreasing intervals. Each iteration consists of one Newton iteration, and the method has an advantage of fast convergence and a natural stopping criterion. This is illustrated for the Fermat–Weber location problem, [2], [3].

REFERENCES

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