

Oral Qualifying Exam Syllabus

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Major Topic: Lie Algebras

1. Basic Concepts
 - (1) Basic Definitions and Examples
 - (2) Solvable, Nilpotent, Simple, and Semisimple Lie Algebras
 - (3) Engel's Theorem
2. Semisimple Lie Algebras and Root Systems
 - (1) Theorem of Lie and Cartan
 - (2) Killing Form, Semisimplicity and Simplicity
 - (3) Weyl's Theorem
 - (4) Representations of $\mathfrak{sl}_2$
 - (5) Root Space Decomposition and Root Systems
 - (6) Simple Roots and Weyl Groups
 - (7) Classification of Simple Lie Algebras over $\mathbb{C}$
3. Representation Theory of Semisimple Lie Algebras
 - (1) Universal Enveloping Algebra and Poincaré-Birkhoff-Witt Theorem
 - (2) Construction via Generators and Relations
 - (3) Theory of Weights
 - (4) Highest Weight Modules, Verma Modules, and Irreducible Finite Dimensional Modules
 - (5) Basic Theory of Multiplicity and Characters

Reference: [1]

Minor Topic: Representation Theory

- Real Forms of Lie Algebras [3]
 1. Definition of Real Form
 2. Cartan Involution and Cartan Decomposition
 3. Restricted Root Systems
 4. Vogan's Diagram and Classification of Real Simple Lie Algebras
- Lie Superalgebras [2]
 1. Definition of Lie Superalgebra
 2. Classical Lie Superalgebras
 3. Classification of Simple Lie Superalgebras with non-degenerate Killing Form
 4. Classification Result of Simple Lie Superalgebras

References

- [1] J.E. Humphreys. *Introduction to Lie Algebras and Representation Theory*. Graduate Texts in Mathematics. Springer New York, 1972. ISBN: 9781461263982.
- [2] V.G. Kac. “Lie superalgebras”. In: *Advances in mathematics* 26.1 (1977), pp. 8–96.
- [3] A.W. Knap. *Lie Groups Beyond an Introduction*. Progress in Mathematics. Birkhäuser Boston, 1996. ISBN: 9781475724530.