Syllabus for Oral Qualifying Exam

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1 Major Topic: Partial Differential Equations

1.1 The Wave Equation

1. Fundamental solutions of linear wave equations.
2. Energy estimates and high order energy estimates.
3. Local well-posedness theory for linear and nonlinear wave equations.
4. Klainerman-Sobolev inequality and its application in wave equations.
5. Global existence theory for wave equations with dimension ≥ 3.
6. Null and weak null conditions.
7. Physical approach to decay for the wave equation: r^p energy estimates and Morawetz estimates.

1.2 The Elliptic Equation

1. Fundamental solutions of Laplace equations.
2. Maximum principle, mean value property, gradient estimates, energy estimate and Harnack inequality for harmonic functions.
3. Existence theory for second order elliptic equations.
4. Hölder continuity of weak solutions.

1.3 Einstein Equations

1. Special solutions of Einstein equations.
2. Local cauchy problem for Einstein vacuum equations.
3. Causal structure.

2 Minor Topic: Analysis

2.1 Functional Analysis

1. Hilbert Space: Riesz lemma, Lax-Milgram theorem and Hilbert basis.
2. Banach Space: Dual space, Hahn-Banach theorem, Baire category theorem and its applications.
3. L^p space: Interpolation, duality, completeness and compactness.
4. Sobolev space: Completeness, duality, compactness, Sobolev embedding and its corollary.
5. Operator theory: Compact operator, self-adjoint operator and spectral theorem for self-adjoint operator.
6. Distribution: Definition, topology, convolution, compactly supported distribution, tempered distribution, and Fourier transformation of distribution.

2.2 Harmonic Analysis

1. Young inequality, Hölder inequality, Hardy-Littlewood-Sobolev inequality and Riesz interpolation.
2. Convolutions and Fourier transform.
3. Homogeneous and inhomogeneous Sobolev space H^s and Fractional Sobolev embedding.
4. Riesz transformation and Poisson Kernel.
5. Calderon-Zygmund kernel and L^p boundness of singular intergal.
6. Mikhlin multiplier theorem, Littlewood-Paley decomposition and Littlewood-Paley square function estimate.

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