ALVIS SYLLABUS FOR ORAL QUALIFICATION

ALVIS DONGHAN ZHAO

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Exam time: 11:00AM, Jan 18th.

1. Major topic: Functional Analysis

1.1. **Hilbert Space.**

(1) The geometry of Hilbert space
(2) The Riesz lemma
(3) Orthonormal bases

1.2. **Banach Space.**

(1) Duals and double duals
(2) The Hahn-Banach theorem
(3) The Baire category theorem

1.3. **Topological Space.**

(1) General notions
(2) Convergence
(3) Compactness
(4) Weak topologies on Banach spaces

1.4. **Bounded Operators.**

(1) Topologies on bounded operators
(2) Adjoints
(3) The spectrum
(4) Compact operators
(5) Spectral theory for compact operators
(6) Fredholm Theorem

1.5. **Unbounded Operators.**

(1) Domains, graphs, adjoints, and spectrum
(2) Symmetric and self-adjoint operators

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2. Minor topic: Partial Differential Equations

2.1. **Laplace's equation.**

(1) Harmonic functions and subharmonic functions.
(2) Weak formulation.
(3) Maximum principle.
(4) Mean value property.
(5) Existence and uniqueness in domains.
(6) Poisson kernel on the half plane.

2.2. **Heat equation.**

(1) Weak formulation.
(2) Maximum principle.
(3) Existence and uniqueness in domains.

2.3. **Wave equation.**

(1) Weak formulation.
(2) Existence and uniqueness in domains.
(3) Domain of dependence.

2.4. **Sobolev Spaces and the Variational Formulation of Elliptic Boundary Value Problems.**

(1) Sobolev spaces $W^{k,p}(\Omega)$.
(2) Extension Operators.
(3) Sobolev Inequalities
(4) The space $W_0^{1,p}(\Omega)$.
(5) Lax-Milgram Theorem.
(6) Variational Formulation of Boundary Value Problems.

References

[1] H. Brezis. *Functional analysis, Sobolev spaces and partial differential equations*. Springer, 2011.
[2] L. C. Evans. *Partial differential equations. Evans*. American Mathematical Society, 1998.
[3] M. Reed and B. Simon. *I: Functional Analysis*. Methods of Modern Mathematical Physics. Elsevier Science, 1981.