

Oral Qualifying Exam Syllabus

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1 Symplectic Geometry

1.1 Symplectic and Lagrangian Manifolds

Symplectic Forms

Cotangent Bundle

Tautological and Canonical Forms in Coordinates

Coordinate-Free Definitions

Naturality of the Tautological and Canonical Forms

Lagrangian Submanifolds of T^*X

Conormal Bundles

1.2 Local Forms

Isotopies and Vector Fields

Tubular Neighborhood Theorem

Homotopy Formula

Moser Relative Theorem

Darboux Theorem

Weinstein Lagrangian Neighborhood Theorem

1.3 Almost Complex Structures and Kähler Manifolds

Complex Structures on Vector Spaces

Compatible Structures

Compatible Triples

Complex Charts

Forms on Complex Manifolds

Kähler Forms

Kähler Potential

The Fubini-Study Structure

1.4 Hamiltonian Systems

Hamiltonian and Symplectic Vector Fields
Classical Mechanics
Brackets
Integrable Systems
Actions
Moment and Comoment Maps
Orbit Spaces

1.5 Symplectic Reduction

Marsden-Weinstein-Meyer Theorem
Noether Principle
Elementary Theory of Reduction
Reduction for Product Groups
Reduction at Other Levels

2 Algebraic Topology

2.1 Homology

Simplicial Homology
Singular Homology
Homotopy Invariance
Exact Sequences and Excision
The Equivalence of Simplicial and Singular Homology
Degree
Cellular Homology
Mayer-Vietoris Sequences

2.2 Cohomology (General Theory)

The Universal Coefficient Theorem
Cohomology of Spaces
Cup Product
The Cohomology Ring
Künneth Formula
Orientations and Homology
Cap Product

2.3 De Rham Cohomology

De Rham Cohomology Groups

Mayer-Vietoris Theorem

Computations

De Rham Theorem

Poincaré Duality on an Orientable Manifold

The Poincaré Dual of a Closed Oriented Submanifold