

Oral Qualifying Exam Syllabus

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1 Algebraic Geometry

1.1 Schemes

1. Sheaves: morphisms, direct image functor, inverse image functor.
2. Schemes: (locally) ringed spaces, affine schemes, Proj of a graded ring, morphisms, functor from the category of varieties over an algebraically closed field k to the category of schemes over k .
3. First properties of schemes: integral, (locally) noetherian schemes, morphisms which are locally of finite type, finite type or finite; open subschemes, closed subschemes, dimension, fiber product of schemes.
4. Separated and proper morphisms: valuative criterion of separatedness, valuative criterion of properness.
5. Sheaves of modules: constructions, locally free sheaves, sheaf of ideals, direct image, inverse image, quasi-coherent sheaf, coherent sheaf, sheaf associated to a graded module, twisted sheaf, very ample invertible sheaf, globally generated sheaf.
6. Divisors: Weil divisor, divisor class group, divisors on curves, Cartier divisor, equivalence of Weil divisor and Cartier divisor, invertible sheaves.
7. Projective morphisms: morphisms to projective space, ample invertible sheaves, linear systems, blowing-up.
8. Differentials: module of relative differentials, sheaves of differentials, nonsingular varieties, applications, some local algebra.
9. Formal schemes: inverse limit of sheaves, formal schemes.

1.2 Cohomology of sheaves

1. Derived functors
2. Cohomology of sheaves: flasque sheaves, a vanishing theorem of Grothendieck.
3. Cohomology of a noetherian affine scheme: Serre's characterization of affine schemes among noetherian schemes.
4. Čech cohomology
5. Cohomology of projective space
6. Ext groups and Ext sheaves
7. Serre Duality: duality for projective space over a field, dualizing sheaf, duality for projective scheme.
8. Higher direct image of sheaves
9. Flat morphisms

- 10. Smooth morphisms
- 11. Theorem on formal functions
- 12. Semicontinuity theorem

1.3 Curves

- 1. Riemann-Roch theorem: Riemann-Roch for curves, special divisor.
- 2. Hurwitz's theorem: ramification divisor, Hurwitz's theorem, Frobenius map.
- 3. Embeddings in projective space
- 4. Elliptic curves: j-invariant, group structure, Jacobian variety, elliptic functions, Hasse invariant, rational points.
- 5. Canonical embedding
- 6. Classification of curves in $\mathbb{P}^3$

2 Homological Algebra

2.1 Tor and Ext

Tor for abelian groups, Tor and Flatness, Ext for nice rings, Ext and extensions, derived functor of the inverse limit, universal coefficient theorems.

2.2 Spectral Sequences

terminology, Leray-Serre spectral sequence, spectral sequence of a filtration, convergence, spectral sequence of a double complex, Hyperhomology, Grothendieck spectral sequence, exact couple.

2.3 Group Homology and Cohomology

definitions and first properties, cyclic and free groups, Shapiro's lemma, crossed homomorphisms and H^1 , the bar resolution, factor sets and H^2 , restriction, corestriction, inflation, transfer, Lyndon-Hochschild-Serre spectral sequence, universal central extensions, covering spaces in topology, Galois cohomology and profinite groups.

References

- [1] R. Hartshorne, *Algebraic Geometry*, Springer-Verlag, New York, 1977.
- [2] C. Weibel, *An Introduction to Homological Algebra*, Cambridge Univ. Press, First paperback edition, 1995.