

Oral Qualifying Exam Syllabus

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1. Experimental Mathematics

(a) Basics

Maple coding, dynamic programming, Newton's method, Goulden-Jackson cluster methods

(b) Permutation

Inversions of permutations, permutations of a multiset, runs, Young tableaux and Robinson-Schensted-Knuth algorithm, hook-length formula, four bases of symmetric functions

2. Combinatorics

(a) Enumeration

Bijections, generating functions, recurrence relations, binomial and multinomial coefficients, inclusion-exclusion, Stirling numbers

(b) Partially Ordered Sets

Hasse diagrams, chains and antichains, Dilworth's theorem, Mirsky's theorem

(c) Hypergraphs

Sperner's theorem, LYM inequality, statement of Kruskal-Katona theorem, Erdos-Ko-Rado theorem, Fisher's inequality, Ray-Chaudhuri-Wilson theorem, Frankl-Wilson theorem(non-uniform R-W)

(d) Ramsey Theory

Ramsey's theorem, infinite Ramsey theorem, Van der Waerden theorem

(e) Probabilistic Methods

Linearity of expectation, Markov's inequality, Chebychev's inequality, Chernoff bounds, alteration methods, second moment method, local lemma

(f) Correlation Inequalities

Four function theorem, FKG inequality, Harris-Kleitman lemma(monotone properties)

3. Graph Theory

(a) Basics

Basic graph definitions, trees, bipartite graphs, cycles and paths

(b) Matching

Konig's theorem, Hall's marriage theorem, Tutte's 1-factor theorem

(c) Connectivity

Menger's theorem, Max-flow min-cut theorem

(d) Planarity

Euler's formula, Kuratowski's theorem, Wagner's theorem, proofs that K_5 and $K_{3,3}$ are nonplanar

(e) Coloring

Five color theorem, vertex coloring(Brooks' theorem), edge coloring(Vizing's theorem)

(f) Extremal Graph Theory

Turan's theorem, statement of Erdos-Stone theorem

4. Probability Theory

(a) Basic Definition

Algebra and sigma-algebra, probability spaces, random variables, probability measure, distribution functions, expectation, moments, independence of random variables, convergence concepts(a.e., in pr., in L^p), probability generating functions, moment generating function

(b) Law of Large Numbers

Weak law of large numbers, strong law of large numbers,

(c) Characteristic Functions and Central Limit Theorem

Definition and basic properties, uniqueness theorem, inversion formula, convergence in distribution, statement of central limit theorem

(d) Mathematical Finance

Change of measure, conditional expectation, conditional independence, martingale, Markov property, Markov process and Markov chain, binomial no-arbitrage pricing model, Brownian motion, quadratic variation, Ito's integral, Ito-Doeblin formula, Black-Scholes formula