

SYLLABUS FOR ORAL QUALIFYING EXAM

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MAJOR TOPIC: PARTIAL DIFFERENTIAL EQUATION

• Laplacian Equation

- (1) Fundamental solution.
- (2) Mean value property.
- (3) Properties of harmonic functions:
Maximum principle, smoothness, estimates on derivatives, Liouville's Theorem, Harnack's inequality, removable singularity, Schwarz' reflection principle.
- (4) Greens functions for a ball and for a half-space.
- (5) The classic Dirichlet problem by Perrons method.
- (6) The Dirichlet Principle by variational method.
- (7) Newtonian potential and the Hölder estimates for its second derivative.

• Heat Equation

- (1) Fundamental Solution.
- (2) Maximum principles on bounded $\bar{U}_T$ and $\mathbb{R}^n \times [0, T]$.
- (3) Backward uniqueness by energy methods.

• Sobolev Space

- (1) Definition of Sobolev space.
- (2) Approximation by smooth functions.
- (3) Extensions and Traces.
- (4) Gagliardo-Nirenberg-Solbolev inequality, Morreys inequality.
- (5) Compact embedding theorem: Rellich-Kondrachov theorem.
- (6) Poincares inequality, Difference quotients.

• Second Order Elliptic Equations

- (1) Definition of weak solutions.

- (2) Existence by Lax-Milgram theorem and energy estimates, Fredholm alternative.
- (3) $W^{2,p}$ Interior regularity and boundary regularity.
- (4) Weak and strong maximum principles, Hopfs lemma, Harnack inequality.
- (5) Eigenvalues and eigenfunctions of symmetric elliptic operator $Lu = -\partial_i(a^{ij}\partial_j u)$.

- **Second Order Parabolic Equations**

- (1) The definition of weak solutions.
- (2) Existence by Galerkin approximations and uniqueness.

MINOR TOPIC: FUNCTIONAL ANALYSIS

- **Banach and Hilbert Space**

- (1) Hahn Banach Theorem.
- (2) Baire category Theorem, uniform boundedness principle, open mapping theorem, inverse mapping theorem and closed graph theorem.
- (3) Duals and double duals.
- (4) Riesz representation theorem.
- (5) Bessel's inequality, Parseval's theorem.

- **Bounded Operator**

- (1) Adjoint of operators.
- (2) Spectral property.
- (3) Compact operator: Fredholm alternative, Riesz Schauder theorem and Hilbert Schmidt theorem.

REFERENCES

- [1] L. Evans, *Partial Differential Equation*, AMS, 1998.
- [2] D. Gilbarg, N. Trudinger, *Elliptic Partial Differential Equations of Second Order*, Springer, 2001.
- [3] M. Reed, B. Simon, *Functional Analysis*, Elsevier Science, 1981.