

Oral Qualifying Exam

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Combinatorics and Graph Theory

- **Basic Enumeration**
counting argument, generating function, binomial and multinomial coefficients, Stirling formula, Inclusion-Exclusion
- **Set System**
Sperner's Theorem, Erdős-Ko-Rado Theorem, Dilworth's Theorem, LYM inequality, Kruskal-Katona Theorem
- **Ramsey Theory**
Ramsey's Theorem, infinite Ramsey theory, van der Waerden's Theorem
- **Discrepancy Theory**
Beck-Fiala Theorem
- **Matching**
Hall's Theorem, König's Theorem, Tutte's Theorem
- **Connectivity**
Menger's Theorem, Kruskal algorithm, max-flow min-cut
- **Coloring**
Brooks' Theorem, Vizing's Theorem, 5-color Theorem
- **Planarity**
Euler's formula, Kuratowski's Theorem, Wagner's Theorem
- **Extremal Graph Theory**
Turán's Theorem, statement of Szemerédi's Regularity Lemma, Erdős-Stone Theorem

Probabilistic Method

- **Basic of Probability**
Markov's inequality, Chernoff bound, linearity of expectation
- **Alterations**
lower bound of diagonal Ramsey numbers, graph with high girth and high chromatic number
- **Second Moment Method**
Chebyshev's inequality, threshold function of containing a fixed graph.
- **Lovasz Local Lemma**
general case of Local Lemma, symmetric case of Local Lemma, application to lower bound of $R(n, n)$
- **Poisson Paradigm**
Janson inequality, $G(n, c/n)$ is triangle-free

Algebraic Method

- **Linear Algebra Method**
odd-even town and even-odd town, Fisher inequality, Kahn-Kalai disproof of Borsuk's conjecture
- **Polynomial Method**
two-distance set, vanishing lemma, Dvir's solution about Nikodym set, Ray-Chaudhuri-Wilson Theorem, Frankl-Wilson Theorem, Explicit Ramsey graphs, Guth-Katz Theorem
- **Nullstellensatz**
Combinatorial Nullstellensatz, Cauchy-Davenport Theorem, Erdős-Heilbronn Theorem

Representation Theory

- Maschke's Theorem
- Characters
- Orthogonality relations
- Structure of the group algebra $K[G]$ when K is an algebraic closed field with zero characteristic
- Irreducible representation of S_n using Young diagrams