

Oral Exam Syllabus

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I Experimental Math

Basics

Dynamic programming, Identities (Trig, Symmetric function), Proof machines, Proof by differentiation.

Hypergeometric Algorithms

Hypergeometric series: definition and canonical form. β and Γ functions. Similar hypergeometric terms.

Identities: Gauss, Kummer, Saalschütz, Dixon, Clausen, Dougall

Algorithms: Sister Celine, Gosper, Zeilberger, Operator Algebra

The Fundamental Theorem

Reference: Ptekovsek, Wilf, and Zeilberger, $A = B$.

Recurrences

Linear: c-Finite, p-Finite, (Non)-Homogenous, Telescoping, Solving.

Non-linear: Catalan Numbers.

II Combinatorics and Graph Theory

II.1 Combinatorics

Enumeration

Bijections, partitioning, binomial/multinomial Coefficients, stirling numbers, inclusion/exclusion, generating functions

Posets and lattices

Hasse diagrams, chains and antichains, Dilworth's theorem

Hypergraphs

LYM inequality, Sperner's theorem, Erdős-Ko-Rado theorem, Van der Waerden's theorem, Schur's theorem, Fisher's inequality, Ray-Chaudhuri, Wilson/Frankl Wilson, statement of Kruskal-Katona

Probabilistic Method

Linearity of expectation, Markov's inequality, Chebyshev's inequality, Chernoff-Hoeffding, method of alterations, 2^{nd} moment method, Lovasz local lemma

II.2 Graph Theory

Basics

Basic graph definitions, trees, bipartite graphs, cycles and paths

Planarity

Euler's formula, Kuratowski's theorem, Wagner's theorem, proofs that K_5 and $K_{3,3}$ are nonplanar

Matchings

König's theorem, Hall's theorem, Tutte's theorem

Graph Algorithms

Max Flow/Min Cut theorem, Kruskal's algorithm, Dijkstra's algorithm

Colorings

Vertex coloring (Brook's Theorem), edge coloring (Vizing's Theorem), statement of weak/strong perfect graph theorem.

Extremal

Ramsey's Theorem, bounds, Infinite Ramsey, Turán's theorem. Erdős-Stone theorem