

Oral Exam Syllabus

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January 2025

1 Major Topic: Partial Differential Equations

1.1 Wave Equations

1. Fundamental Solution of Linear Wave Equations
2. Energy Estimate for Wave Equations
3. Klainerman-Sobolev Inequality
4. Local Well-posedness for Wave Equations
5. Global Existence of Wave Equation for Dimension ≥ 4 and Dimension 3
6. Null and Weak Null condition
7. r^p energy estimate

1.2 Laplace Equation

1. Fundamental Solution of Laplace Equations, Poisson Kernel
2. Maximum Principle, Mean-Value Formula
3. Energy Estimate

1.3 Variational Methods

1. Euler-Lagrange Equation, Dirichlet Principle
2. Second Variation
3. Existence of Minimizers, Weak Solutions of Euler-Lagrange Equation

2 Minor Topic: Functional Analysis

2.1 Banach Space

1. Hahn-Banach Theorem
2. Duality, Weak and Weak* Topology, Banach-Alaoglu Theorem, Compactness
3. Baire Category Theorem
4. L^p space: Duality, Completeness, Compactness

2.2 Hilbert Space

1. Definition, Dual space, Riesz Representation Theorem
2. Lax-Milgram Theorem
3. Orthonormal Basis

2.3 Sobolev Space

1. Definition and basic properties
2. Sobolev Embedding Theorem, Rellich-Kondrachov Theorem
3. The space $W_0^{1,p}(\Omega)$

2.4 Operator Theory

1. Bounded Operator, Compact Operator
2. Fredholm Alternative
3. Spectrum of Compact Operator
4. Self-Adjoint Operator, Spectral Theorem

2.5 Theory of Distribution

1. Definition, Basic Properties
2. Convergence of Distribution
3. Fourier transform of distribution, tempered distribution

References

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- [2] Lawrence C Evans. *Partial differential equations*, volume 19. American Mathematical Society.
- [3] Peter D Lax. *Functional analysis*. John Wiley & Sons.
- [4] Jonathan Luk. Introduction to nonlinear wave equation.
- [5] Wang Qian. Lectures on nonlinear wave equation.
- [6] Hans Ringström. *The Cauchy problem in general relativity*. European Mathematical Society.