

Oral Qualifying Exam Syllabus

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1. Major Topic: Set Theory

(a) Basic Set Theory

- inductive formula for cardinal arithmetic
- clubs and stationary sets, **SCH** cannot fail for the first time at singular cardinal of uncountable cofinality
- elementary submodels, applications to combinatorics, e.g., Ramsey's theorem
- constructible hierarchy, $\text{Con}(\text{GCH})$, construction of $\diamond$ -sequence.

(b) Forcing

- Cohen forcing, Lévy collapse, shooting a club, etc.
- large cardinals and lifting, getting $2^\kappa > \kappa^+$ for measurable κ
- iterated forcing and $\text{Con}(\text{MA})$, proper forcing and $\text{Con}(\text{PFA})$
- Prikry forcing and $\text{Con}(\neg\text{SCH})$

(c) Infinitary Combinatorics

- Aronszajn and Suslin trees at ω_1 .
- square-like principles, definitions and implications between them
- use forcing to create or thread a square sequence, failure of squares above very large cardinals
- approachability, scales, existence of scale at singular cardinal, Shelah trichotomy
- pcf theory, exact upper bounds, generators, application to cardinal arithmetic
- stationary reflection from ω many supercompacts

2. Minor Topic: Descriptive Set Theory

(a) Basics

- examples of Polish spaces, Alexandrov's theorem, Borel hierarchy
- meager sets, Baire category theorem, Baire property, Kuratowski-Ulam theorem
- analytic separation theorem, regularity properties of analytic sets
- injective image of Borel set, Borel isomorphism

(b) Determinacy

- regularity properties and determinacy
- measurable cardinal implies analytic determinacy

(c) Invariant descriptive set theory

- Mycielski's theorem
- selector, transversal, smoothness, the case of countable Borel equivalence relation
- generic ergodicity implies not smooth