Oral Qualifying Exam Syllabus

Xiaoxu WU

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Major Topic: Functional Analysis

1. Preliminaries
   (a) Sets and functions
   (b) Metric and normed linear spaces
   (c) The Lebesgue integral
   (d) Abstract measure theory
   (e) Two convergence arguments
   (f) Equicontinuity
2. Hilbert spaces
   (a) The geometry of Hilbert space
   (b) The Riesz lemma
   (c) Orthonormal bases
   (d) Tensor products of Hilbert spaces
   (e) Ergodic theory: an introduction
3. Banach spaces
   (a) Definition and examples
   (b) Duals and double duals
   (c) The Hahn-Banach theorem
   (d) Operations on Banach spaces
   (e) The Baire category theorem and its consequences
4. Topological spaces
   (a) General notions

   (b) Nets and convergence
   (c) Compactness
   (d) Measure theory on compact spaces
   (e) Weak topologies on Banach spaces

5. Locally convex spaces
   (a) General properties
   (b) Fréchet spaces
   (c) Functions of rapid decease and the tempered distributions
   (d) inductive limits: generalized functions and weak solutions of partial differential equations

6. Bounded operators
   (a) Topologies on bounded operators
   (b) Adjoints
   (c) The spectrum
   (d) Positive operators and the polar decomposition
   (e) Compact operators
   (f) The trace class and Hilbert-Schmidt ideals

7. The spectral theorem
   (a) The continuous functional calculus
   (b) The spectral measures
   (c) Spectral projections
   (d) Ergodic theory revisited: Koopmanism

8. Unbounded operators
   (a) Domains, graphs, adjoints, and spectrum
   (b) Symmetric and self-adjoint operators: the basic criterion for self-adjointness
   (c) The spectral theorem
   (d) Stone's theorem
   (e) Formal manipulation is a touchy business: Nelson's example
   (f) Quadratic forms
   (g) Convergence of unbounded operators
   (h) The Trotter product formula

Reference: [1]

Minor Topic: Dispersive estimates, Strichartz estimates and applications to nonlinear equations

1. Ordinary differential equations
   (a) General theory
   (b) Gronwall's inequality
   (c) Bootstrap and continuity arguments
   (d) Noether's theorem
   (e) Monotonicity formulae
   (f) Linear and semilinear equations
   (g) Completely integrable systems
2. Constant coefficient linear dispersive equations
   (a) The Fourier transform
   (b) Fundamental solution
   (c) Dispersion and Strichartz estimates
   (d) Conservation laws for the Schrödinger equation
3. Semilinear dispersive equations
   (a) Local existence theory
   (b) Conservation laws and global existence
   (c) Decay estimates
   (d) Scattering theory

Reference: [2]

References

[1] Reed, M., & Simon, B. (1980). Functional analysis. Revised and Enlarged Edition.

[2] Tao, T. (2006). Nonlinear dispersive equations: local and global analysis (No. 106). American Mathematical Soc..