Oral Qualifying Exam Syllabus

Ben-Zion Weltsch

I) Set Theory (main topic)

Forcing and Large Cardinals

A) Basic forcing notions: Cohen, Levy collapse, Prikry, shooting a club, adding a $\square$ sequence, adding a non-reflecting stationary set

B) General forcing results: properties of Prikry sequences, properties of Cohen reals, adding Suslin trees, failure of SCH from supercompact, Solovay's model, GCH cannot first fail at a singular cardinal of uncountable cofinality, Solovay's splitting theorem

C) Iterated forcing: Easton forcing, consistency of MA, PFA, no Suslin trees, Laver indestructible supercompact cardinal, failure of GCH at a measurable, consistency of $TP_{\aleph_2}$

D) Consequences of forcing axioms: No Suslin trees under MA, PFA kills $\aleph_2$ Aronszajn trees + $\square_\kappa$

E) Large cardinal properties: inaccessible, (weak, strongly, super)compact, measurable, strong, Woodin, superstrong, extendible cardinals, etc

Infinitary Combinatorics

A) Combinatorics in ZFC: Delta System lemma, Aronszajn trees, club and stationary sets, ultrafilters

B) Combinatorics in L: GCH, $\diamondsuit$, Suslin trees

C) Stationary reflection and $\square_\kappa$: if every stationary subset of $\omega_2 \cap \mathrm{Cof}(\omega)$ reflects then ω_2 is inaccessible in L, $\square_{\omega_1}$ implies a stationary subset of $\omega_2 \cap \mathrm{Cof}(\omega)$ does not reflect, stationary reflection at innacessible, weakly compact, and measurable cardinals

Inner Models

A) Iterated ultrapowers: wellfoundedness, Mitchell order, comparison, linear iteration of extenders, inner models up to one strong cardinal; $L[U], L[\mathcal{U}], L[\mathcal{E}]$

B) Consequences of the covering lemma and $0^{\#}$, properties and equivalent definitions of $0^{\#}$

II) Descriptive Set Theory (minor topic)

Classical DST and Borel Equivalence Relations

A) Examples of Polish spaces, Alexandroff's theorem, Borel hierarchy

B) Baire category, Kuratowski-Ulam

C) Analytic separation, regularity properties of analytic sets, universal analytic set

D) Borel Equivalence relations: Feldman-Moore, characterizations of smoothness

Determinacy

A) Non-determined set from choice

B) Gale-Stewart

C) Analytic determinacy from a measurable and equivalence with $0^{\#}$

D) Regularity properties of determined sets

E) Consequences of AD: Lebesgue measurability, property of Baire, perfect set property, measurability of ω_1