ORAL QUALIFYING EXAM SYLLABUS

YUCHEN WEI

•Major Topic: Symmetric Functions

(1) The ring of symmetric functions Λ.

- Elementary symmetric functions e_r.
- Complete symmetric functions h_r.
- Power sums p_r.

How to prove

$$\Lambda = \mathbb{C}[e_0, e_1, e_2, \ldots] = \mathbb{C}[h_0, h_1, h_2, \ldots] = \mathbb{C}[p_0, p_1, p_2, \ldots]$$

How to express e_r, h_r, p_r in terms of each other.

(2) Partitions and semistandard tableaux, Robinson-Schensted-Knuth bijection, row and column insertions, plactic monoid, connection with increasing subsequences.

(3) Schur functions S_λ. Equivalent definitions

- $\sum_T x^T$
- $\det\left(x_i^{\lambda_j+n-j}\right) / \prod_{1\le i<j\le n}(x_i - x_j)$

(4) Complex representations of S_n.

(5) Schur functions from the representation theory of S_n and of $GL_n(\mathbb{C})$, Schur-Weyl dulaity.

(6) Cauchy identity, Pieri formulas, Jacobi-Trudi formulas, Hook length formula, Weyl dimension formula.

(7) The Littlewood-Richardson rule, skew Schur functions.

(8) Hall-Littlewood symmetric functions, connections to representation theory of GL_n over a finite field and Hecke rings.

(9) Macdonald symmetric functions. Operators D_n^r. Connections to double affine Hecke algebras.

(10) The (q,t)-Kostka coefficients $K_{\lambda\mu}(q,t) \in \mathbb{Z}[q,t]$ is a polynomial in q and t with integral coefficients.

•Minor Topic: Lie Algebras

(1) Basic theory of Lie algebras.

- Definitions, representations, modules
- Solvable, nilpotent, semisimple Lie algebras
- Engel's theorem

(2) Semisimple Lie algebras.

- Lie's theorem and Cartan's criterion

- Killing form
- Weyl's theorem

(3) Root systems.

- Definition of root systems
- Simple roots and the Weyl group
- Classification of root systems

(4) Existence Theorem.

- Universal enveloping algebras
- Poincaré-Birkhoff-Witt theorem
- Serre's theorem

References

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