Oral Exam Syllabus

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1 Major: Partial Differential Equations

1.1 Laplace's Equation

(Refer to [2] section 2.8 for sub and super harmonic functions and Perron's method and [1] section 2.2 for the rest)

- Fundamental solution, Poisson's equation
- Mean-value formulas
- Properties of harmonic functions: Maximum principles, sub and super harmonic functions, Perron's method
- Green's function
- Energy methods

1.2 Heat Equation

(Refer to [1] section 2.3)

- Fundamental solution
- Mean-value formula
- Properties of solutions, maximum principles
- Energy methods

1.3 Sobolev Spaces

(Refer to [1] section 5.1-5.7 and 5.8.1)

- Weak derivatives
- Definition and elementary properties

- Approximation by smooth functions
- Extensions
- Traces
- Sobolev inequalities: Gagliardo-Nirenberg-Sobolev inequality, Morrey's inequality, General Sobolev inequalities
- Compactness: Rellich-Kondrachov compactness theorem, Arzela-Ascoli compactness criterion
- Poincaré's inequalities

1.4 Second Order Elliptic Equations

(Refer to [1] section 6.1-6.4)

- Definitions of elliptic equations and weak solutions
- Existence: Lax-Milgram theorem, energy estimates, Fredholm alternative
- Regularity
- Maximum principles: Weak maximum principle, Hopf's lemma, strong maximum principle
- Harnack's inequality

1.5 Other Definitions and Statements (without Proof)

- Hölder spaces
- Schauder estimates
- The continuity method
- Calderon-Zygmund inequality
- viscosity solution

2 Minor: Riemannian Geometry and Basics of Minimal Surfaces

2.1 Riemannian Geometry

(Based on the differential geometry course taught by Professor Daniel Ketover)

- Affine connection, Levi-Civita connection

- Geodesics, parallel transport, minimizing properties, exponential map, normal coordinates, first variation formula
- Hopf-Rinow theorem
- Sectional curvature, Ricci curvature, scalar curvature
- Jocobi fields, conjugate points
- The second variation formula, the Bonnet-Myers theorem
- Isometric immersions, second fundamental form

2.2 Basics of Minimal Surfaces

(Refer to [6] section 1.1-1.8)

- Minimal surface equation
- First variation formula
- Bernstein's theorem
- The strong maximum principle
- Second variation formula

References

[1] L. C. Evans, *Partial Differential Equations, 2nd edition.* AMS, 2010.

[2] D. Gilbarg, N. S. Trudinger, *Elliptic Partial Differential Equations of Second Order.* Springer, 2001.

[3] J. Jost, *Partial Differential Equations, 3rd edition.* Springer, 2013.

[4] M. do Carmo, *Riemannian Geometry.* Birkhäuser Boston, 1992.

[5] J. M. Lee *Introduction to Riemannian Manifolds, 2nd edition.* Springer, 2018.

[6] T. H. Colding, W. P. Minicozzi, *A Course in Minimal Surfaces.* AMS, 2011.