

ORAL QUALIFYING EXAM SYLLABUS

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1. SCHEMES

1. **Sheaves:** Presheaves and associated sheaves, morphisms of sheaves, stalks, pull-back and push-forward
2. **Schemes:** Affine schemes, Proj, structure sheaf, scheme associated to a variety, morphisms of schemes, generic points
3. **First Properties of Schemes:** Reduced, integral, Noetherian schemes, morphisms of finite type, finite morphisms, closed immersions, dimension, fiber products
4. **Separated and Proper Morphisms:** Definition of separated and proper morphisms, valuative criteria, projective morphisms, scheme-theoretic image
5. **Coherent and Quasi-coherent Sheaves:** Definition of $\mathcal{O}_X$ -modules, quasi-coherent and coherent sheaves, constructions on $\mathcal{O}_X$ -modules, invertible sheaves, vector bundles
6. **Divisors:** Weil divisors, Cartier divisors, equivalence for locally factorial schemes, Picard groups
7. **Projective Morphisms:** Criteria and characterization of projective morphisms, ample and very ample line bundles, blowups
8. **Differentials:** Derivations, module of relative differential forms, sheaves of differentials and nonsingularity, tangent sheaf, canonical sheaf, geometric genus

2. COHOMOLOGY

1. **Derived Functors:** Abelian categories, complexes, derived functors, δ -functors
2. **Cohomology of Sheaves:** The category of sheaves of $\mathcal{O}_X$ -modules has enough injectives, Grothendieck's vanishing theorems
3. **Cohomology of a Noetherian Affine Scheme:** vanishing of higher cohomology of quasi-coherent sheaves and Serre's criterion for being affine in terms of vanishing of cohomology
4. **Cech Cohomology:** Definition, isomorphism with regular cohomology for a Noetherian separated scheme
5. **Cohomology of Projective Spaces:** Calculation using Cech cohomology, Serre vanishing, cohomological criterion of ampleness

3. COMPLEX GEOMETRY

1. **Several Complex Variables:** Cauchy's formula and applications, Hartogs' theorem, Weierstrass' theorems, analytic varieties

2. **Complex Manifolds:** Sub-manifolds and Sub-varieties, De Rham and Dolbeault cohomology, calculus on complex manifolds, holomorphic vector bundles

3. **Kahler Manifolds:** Hodge theorem and applications, the Kahler condition, the Hodge Identities, and the Hodge decomposition

REFERENCES

- [1] R. Hartshorne, *Algebraic Geometry*, Springer-Verlag, 1977.
- [2] Griffiths P, Harris J. *Principles of algebraic geometry*, [M]. New York: Wiley, 1978.