

Oral Qualifying Exam Syllabus

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Algebraic Geometry

1. **Sheaves:** Definition, presheaves and associated sheaves, morphisms, stalks, pull-back and push-forward
2. **Schemes:** Affine schemes, Proj, structure sheaf, schemes associated to varieties
3. **First properties of schemes:** Reduced, integral, Noetherian schemes, morphisms of finite type, finite morphisms, closed immersions, dimension, fibre product
4. **Separated and proper morphisms:** Definition of separated and proper morphisms, valuative criteria, projective morphisms, reduced structure of closed subset, scheme-theoretic image, constructible sets
5. **Coherent and quasi-coherent sheaves:** Definition of $\mathcal{O}_X$ -modules, quasi-coherent and coherent sheaves, construction of $\mathcal{O}_X$ -modules, invertible sheaves, vector bundles
6. **Divisors** Weil divisors, Cartier divisors, equivalence for locally factorial schemes, invertible sheaves
7. **Projective morphism:** Criteria and characterization of projective morphisms, ample and very ample sheaves, blowups
8. **Differentials:** Derivations, module of relative differential forms, sheaves of differentials and connection to nonsingularity, tangent sheaf, canonical sheaf, geometric genus

Homological Algebra

1. **Derived functors:** Abelian categories, complexes, derived functors, δ -functors
2. **Cohomology of sheaves:** The category of sheaves of $\mathcal{O}_X$ -modules has enough injectives
3. **Cohomology of noetherian affine schemes:** Characterization of noetherian affine schemes by cohomology of sheaves
4. **Čech cohomology:** Definition, isomorphism with sheaf cohomology for a noetherian separated scheme
5. **Cohomology of projective space**
6. **Serre duality theorem**
7. **Spectral sequence:** Filtered complex, double complex
8. **Hypercohomology**

Toric Varieties

1. **Definitions:** Convex polyhedral cones, geometry of convex sets, affine toric varieties, fans and toric varieties, toric varieties from polytopes
2. **Singularities and compactness:** Local properties of toric varieties, surfaces, quotient singularities, one-parameter subgroups, limit points, compactness and properness, nonsingular surfaces, resolution of singularities
3. **Orbits, topology, and line bundles:** Orbits, fundamental groups and Euler characteristics, divisors, line bundles, cohomology of line bundles, canonical class
4. **Cohomology of smooth toric varieties:** Stanley-Reisner relations, self-intersection of invariant divisors

References

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- [3] C. Weibel, An Introduction to Homological Algebra, Cambridge Studies in Advanced Mathematics, 1995
- [4] W. Fulton, Introduction to Toric Varieties, Annals of Math Studies, 1993