

Qual Syllabus

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Elliptic PDE

Harmonic Functions/ Poisson Equation

- Mean value property: maximum principle, regularity, derivative estimates, Harnack Inequality
- Fundamental Solution: Green's Representation Formula, Poisson Kernel, Schauder Estimates for Poisson Equation, removable singularities
- Perron's Method
- Dirichlet Principle: Direct Method of Calculus of Variations
- Bernstein's method for gradient estimates

Non-Divergence Form Elliptic Equations

- Weak maximum principle: C^0 apriori estimate for Dirichlet problem
- Hopf lemma, strong maximum principle

Sobolev Spaces

- Definitions
- Density of smooth functions: chain rule, product rule
- Trace theorem
- Extensions
- Sobolev Inequalities: Gagliardo-Nirenberg, Morrey, Poincare
- Embeddings/ compact embeddings
- Difference Quotients
- Interpolation Inequalities

Weak Solutions to Divergence Form Equations

- Existence: Lax Milgram + Fredholm Alternative
- L^2 regularity (Difference Quotients)
- Caccioppoli Inequality
- De-Giorgi's Proof of L^∞_{loc} and $C^{0,\alpha}_{loc}$ regularity

Functional Analysis

Banach Space:

- Hahn-Banach
- Dual space
- Reisz Lemma
- Weak topology, weak-* topology: Banach Alaoglu
- Reflexivity: Kakutani's Theorem

Hilbert Space:

- Cauchy-Schwarz, parallelogram law
- Reisz Representation Theorem: Lax-Milgram Lemma
- Hilbert Basis, Bessel's Inequality

Bounded Operators:

- Baire Category Theorem: Uniform boundedness, open mapping, closed graph theorems
- Adjoint Operator
- Compact operators
- Fredholm Alternative
- Spectrum of compact self adjoint operator