Oral Qualifying Exam Syllabus

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Modular Forms

(i)Hecke operators. Commutativity and self-adjointness with respect to Petersson inner product. Dimension of $M_k(\Gamma)$ for $\Gamma = SL(2,\mathbb{Z})$.
(ii)Fourier expansion of modular forms especially Eisenstein series.
(iii)Poincare Series for $SL(2,\mathbb{Z})$.
(iv)Non-holomorphic Eisenstein series and the Rankin-Selberg method.
(v)The Jacobi Θ function, Δ function, and j-function.
(vi)L-functions of modular forms and functional equations.

Analytic Number Theory

(i)Dirichlet characters, primitive characters, and Gauss sums.
(ii)Dirichlet's theorem on primes in arithmetic progressions. Landau lemma.
(iii)Poisson summation. Mellin transform.
(iv)Riemann Zeta function and Dirichlet L-functions. Their analytic continuation, Euler product, Hadamard product, explicit formula, and convexity bound.
(v)Prime number theorem for primes in arithmetic progressions.

Algebraic Number Theory

(i)Number fields. Their ring of integers and integral bases.
(ii)Real and complex embeddings. The Galois group. Galois extensions, discriminant, and trace form.
(iii)Fractional ideals. Prime ideals, unique factorization, ramification and splitting. Norm of elements and ideals.
(iv)Dirichlet's unit theorem, the class group and Minkowski's bound.
(v)Dedekind Zeta function and class number formula.

References

[1]Davenport, H. *Multiplicative Number Theory.* New York: Springer, 1980.
[2] Iwaniec, H. *Topics in Classical Automorphic Forms.* Providence, R.I.: American Mathematical Society, 1997.
[3] Iwaniec,H and Kowalski,E. *Analytic Number Theory*, American Mathematical Society, 2004.
[4] Marcus, D.A. *Number Fields* New York: Springer, 1977.
[5]Serre, J-P. *A Course in Arithmetic* New York: Springer, 1973