

Oral Qual Syllabus

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1 Major Topic: Combinatorics

1. **Enumeration:** counting and double counting, pigeonhole principle, inclusion-exclusion, Stirling's approximation, binomial identities, Catalan numbers, generating functions
2. **Set systems:** Sperner's theorem, Lubell-Yamamoto-Meshalkin and Bollobás' inequality, Erdős-Ko-Rado theorem, Kruskal-Katona and Harper's theorem, Fisher's inequality and odd/even town, Ray-Chaudhuri Wilson theorem, Frankl-Wilson theorem, statement of Baranyai's theorem
3. **Posets and lattices:** Dilworth's theorem (on partition into chains), Birkhoff's representation theorem, Harris-Kleitman, Fortuin-Kasteleyn-Ginibre, Ahlswede-Daykin, statement of Sahi's theorem and conjecture, Rinott-Saks 2m-function theorem, van den Berg-Kesten inequality, statement of Reimer's theorem, Linear extensions: Shepp's theorem $\{x < y\} \uparrow \{x < z\}$; counterexample to $\{w < x < z\} \uparrow \{w < y < z\}$
4. **Graph Theory:**
 - Matchings: König's Theorem, Hall's Matching Theorem, Tutte's Theorem
 - Connectivity: Kruskal and Prim spanning tree algorithms, Menger's Theorem, maxflow min-cut
 - Planarity: Euler's formula, Kuratowski's theorem
 - Coloring: Five-Color Theorem, Brooks' theorem, Vizing's theorem, Thomassen's Theorem on list-coloring of planar graphs, Sperner's lemma
 - Extremal: Mantel's Theorem, Turán's Theorem, Erdős-Stone-Simonovits Theorem, statement of Szemerédi's Regularity Lemma
5. **Ramsey Theory:** Ramsey's theorem for graphs and hypergraphs, infinite Ramsey, van der Waerden's theorem, Hales-Jewett theorem, Rado's theorem, statement of upper bound on diagonal Ramsey numbers

6. Probabilistic Methods:

- Basics: linearity of expectation, union bound, Markov's inequality, Chernoff bounds
- Alterations: lower bound for diagonal Ramsey numbers, graph with high girth and high chromatic number
- Second Moment Method: Chebyshev's inequality, threshold function to contain a fixed subgraph
- Lovász Local Lemma: statement of symmetric and general versions, application to lower bounds for diagonal Ramsey numbers
- Martingales: definition, statement of Azuma's inequality, statement of Doob's martingale convergence theorem

2 Minor Topic: Experimental Math

1. Basics
2. Algorithms for Game Theory