

Oral Qualifying Exam Syllabus

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Main Topic: Algebraic Geometry

• Schemes

1. **Sheaves:** Definition, presheaves and associated sheaves, morphisms, stalks, pull-back and push-forward.
2. **Schemes:** Affine schemes, Proj, structure sheaf, scheme associated to a variety
3. **First properties of schemes:** Reduced, integral, Noetherian schemes, morphisms of finite type, finite morphisms, closed immersions, dimension, fibre products
4. **Separated and proper morphisms:** Definition of separated and proper morphisms, valuative criteria, projective morphisms, reduced structure of closed subsets, scheme-theoretic image, constructible sets
5. **Coherent and quasi-coherent sheaves:** Definition of $\mathcal{O}_X$ -modules, quasi-coherent and coherent sheaves, constructions on $\mathcal{O}_X$ -modules, invertible sheaves, vector bundles
6. **Divisors:** Weil divisors, Cartier divisors, equivalence for locally factorial schemes, invertible sheaves
7. **Projective morphisms:** Criteria and characterization of projective morphisms, ample and very ample line bundles, blowups
8. **Differentials:** Derivations, module of relative differential forms, sheaves of differentials and nonsingularity, tangent sheaf, canonical sheaf, geometric genus

• Cohomology

1. **Derived functors:** Abelian categories, complexes, derived functors, δ -functors
2. **Cohomology of sheaves:** The category of sheaves of $\mathcal{O}_X$ -modules has enough injectives, some basic vanishing theorems
3. **Cohomology of a Noetherian Affine Scheme:** Vanishing of higher cohomology of quasi-coherent sheaves and Serre's criterion for being affine in terms of vanishing of cohomology

4. **Čech Cohomology:** Definition, isomorphism with regular cohomology for a noetherian separated scheme
5. **Cohomology of projective space:** Calculation using Čech cohomology, Serre vanishing, cohomological criterion of ampleness
6. **Ext Groups and sheaves:** Definition and basic properties
7. **Serre duality:** Dualizing sheaf and duality for a projective scheme, invertibility of dualizing sheaf for local complete intersection, isomorphism between dualizing sheaf and canonical sheaf for nonsingular projective variety

- **The Geometry of Flag Varieties (of Type A)**

1. Grassmannians and the Plucker Embedding
2. Flag Varieties, Bruhat Decomposition and Schubert Varieties
3. Schubert Classes, Kleiman's transversality, and Intersections
4. Line Bundles and Characters, Borel-Weil
5. Singularities of Schubert Varieties: Normality, rationality of singularities, cohomology of line bundles
6. The Grothendieck Ring and a Degeneration of the Diagonal of a Flag Variety
7. Positivity of the Structure Constants of the Grothendieck Ring

Minor Topic: Lie Algebras

- **Basic concepts of Lie Algebras**

1. Ideals, Homomorphisms, Representations, and Derivations
2. Solvable and Nilpotent Lie Algebras
3. Engel's theorems

- **Semisimple Lie Algebras**

1. Lie's Theorem and Cartan's Criterion
2. Killing Form
3. Completely Reducible Representations and Weyl's Theorem
4. Root Systems
5. Simple Roots and Weyl Group
6. Classification of (Semi)simple Lie Algebras

- **PBW and Existence theorems**

1. Universal Enveloping Algebras
2. Poincaré-Birkhoff-Witt Theorem
3. Serre's Theorem