

Oral Qualifying Exam Syllabus

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Main Topic: Representation Theory of Finite Groups

1. Group Algebras

- (1) Schur's Lemma
- (2) Wedderburn's Theorem

2. Complex Representations and Characters

- (1) Schur Orthogonality Relations
- (2) Integrality of Characters and Central Characters
- (3) Tensor Products
- (4) Frobenius-Schur Indicator

3. Induction and Restriction

- (1) Frobenius Reciprocity
- (2) Clifford's Theorem
- (3) Clifford Correspondence and Gallagher's Theorem

4. Representations over Subfields of $\mathbb{C}$

- (1) Absolutely Irreducible Representations and Splitting Fields
- (2) Schur Index

5. Projective Representation

- (1) Central Extensions
- (2) Schur Multiplier
- (3) Character Triples

Minor Topic: Lie Algebras

1. Basic Concepts of Lie Algebras

- (1) Ideals, Homomorphisms, Representations, and Derivations
- (2) Solvable and Nilpotent Lie Algebras
- (3) Engel's Theorem

2. Semisimple Lie Algebras

- (1) Lie's Theorem and Cartan's Criterion
- (2) Killing Form
- (3) Completely Reducible Representations and Weyl's Theorem
- (4) Root Systems
- (5) Simple Roots and Weyl Group
- (6) Classification of (Semi)simple Lie Algebras (existence and uniqueness)

3. Representation Theory

- (1) Universal Enveloping Algebras
- (2) Poincaré–Birkhoff–Witt Theorem
- (3) Serre's Theorem

References

- 1. I. M. Isaacs, *Character Theory of Finite Groups*, Academic Press, New York, 1976. MR0460423.
- 2. J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory*, Graduate Texts in Mathematics, 9, Springer-Verlag, New York, 1978. MR0499562.