Oral Qualifying Exam Syllabus

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1 Experimental Mathematics

1. Maple skills
   – Maple Syntax
   – Dynamic Programming

2. Basics
   – Selection sort and Quicksort algorithms
   – Recurrence equations
   – Symbolic Sums
   – Generating Functions
   – Asymptotic Estimates

3. Formal Power Series
   – Differentiation and Division
   – Sequences of Power Series
   – The Transfer Principle
   – Multivariate Power series
   – Truncated Power Series

4. Polynomials
   – Polynomials as power series
   – Polynomials as sequences
   – The Tetrahedron for polynomials: symbolic sums, asymptotic estimates, generating functions, recurrence relations
   – Polynomials as solutions
   – Polynomials as coefficients

5. C-Finite Sequences
   – Fibonacci Numbers
   – Recurrences with Constant Coefficients
   – Closure Properties
   – The Tetrahedron for C-finite sequences: symbolic sums, asymptotic estimates, generating functions, recurrence relations
   – System of C-finite Recurrences

2 Graph Theory

1. Matchings
   – Konig's Theorem
   – Hall's Theorem
   – Tutte's Theorem

- Stable matchings
- Matching polytopes

2. Connectivity
   - Menger's Theorem
   - Max Flow-Min Cut Theorem
   - Kruskal's Algorithm
   - Prim's Algorithm
   - Dijkstra's Algorithm
   - Matrix-Tree Theorem
   - Cayley's Formula

3. Coloring
   - 5-color theorem
   - Brooks Theorem
   - Vizing's Theorem
   - List colorings
   - Ramsey Theory

4. Extremal Graph Theory
   - Turan's Theorem
   - Zarankiewicz's Theorem
   - Regularity lemma
   - Erdos-Stone theorem
   - Triangle removal lemma
   - Roth's theorem