

Oral Qualifying Exam Syllabus

Qiyuan Song

1. Major: Mean curvature flow

1.1 Basic properties

- Definition of mean curvature flow
- Special solutions
- Short time existence
- Evolution of geometric quantities
- Avoidance principle
- Maximum principle
- Characterization of singular time

1.2 Convexity

- Preservation of convexity
- Convexity and cylindrical estimate in 2-convex case
- Huisken's theorem

1.3 Type I singularity

- Monotonicity formula
- Gaussian density
- Gleaning out lemma
- Rescaling procedure around type I singularity
- Classification of mean convex self-shrinkers

1.4 Type II singularity

- Hamilton's blow up procedure
- Differential Harnack inequality
- Blow up limits of mean convex solutions

2. Minor: Partial differential equations

2.1 Sobolev spaces

- Approximation by smooth functions
- Trace operator
- Sobolev inequalities
- Rellich-Kondrachov compact embedding

2.2 Laplace equations

- Mean value formula, smoothness of harmonic functions
- Fundamental solution
- Representation formula for Poisson's equation
- Energy method
- Gradient estimate, Liouville's Theorem

2.3 Heat equations

- Mean value formula
- Fundamental solution

- Representation formula
- Maximum principles

2.4 Second order elliptic equations

- Strong/weak maximum principles
- Harnack's inequality
- Existence of weak solutions
- L^2 regularity theory

References

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- [2] K. Ecker (2004) Regularity theory for mean curvature flow, vol. 57. Springer Science & Business Media
- [3] L. C. Evans (2010) Partial Differential Equations, 2nd Edition, American Mathematical Society