

Oral Qualification Syllabus

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Major: Complex Differential Geometry

Complex Geometry

From Huybrechts, “Complex Geometry: An Introduction”, Chapters 1–5

- Complex Manifold; Line bundles and Divisors, Blow-ups
- Kähler Manifolds; Kähler Identities, Basic Hodge Theory
- Vector Bundles; Connections, Curvature, Chern Classes, Kähler-Einstein Metrics
- Cohomology Theory; Hirzebruch-Riemann-Roch Theorem, Kodaira Vanishing and Embedding Theorem

Riemannian Geometry

From Do Carmo, “Riemannian Geometry”, Chapters 1–11

- Riemannian Manifolds; Connections, Geodesics, Curvatures
- Jacobi Fields and Comparisons; Hopf-Rinow Theorem, Hadamard Theorem, Rauch Comparison Theorem, Toponogov Theorem, Conjugacy Points, Morse Index Theorem
- Isometric Immersion; Second Fundamental Form, Theorema Egregium
- Spaces of Constant Sectional Curvatures; Classification, Liouville Theorem
- Variation on Energy; First and Second Variations, Bonnet-Myers and Synge-Weinstein Theorem

Minor: Algebraic Topology

From Hatcher, “Algebraic Topology”, Chapters 1–3

- Fundamental Groups; Van Kampen’s Theorem, Covering Spaces and their Galois Connection, Deck Transformation
- Homology Theory; Simplicial, Singular, and Cellular Homology, Short and Long Exact Sequences, Excision, Mayer-Vietoris Sequence, Axiomatic Homology Theory
- Cohomology Theory; Universal Coefficient Theorem, Cohomology Ring, Künneth Formula, Čech Cohomology, Poincaré Duality

References

- [1] Daniel Huybrechts, *Complex Geometry: An Introduction*, Volume 78, Springer, 2005.
- [2] Manfredo P. do Carmo, *Riemannian Geometry*, Volume 2, Springer, 1992.
- [3] Allen Hatcher, *Algebraic Topology*, Cambridge University Press, 2002.