

ORAL EXAM SYLLABUS

SUMIT K SINGH

1. Algebraic Geometry

1.1. Schemes.

- (1) **Sheaves:** Definition, presheaves and associated sheaves, morphisms, stalks, pull-back and push-forward
- (2) **Schemes:** Affine schemes, Proj, structure sheaf, schemes associated to varieties
- (3) **First Properties of Scheme:** Reduced, integral, Noetherian schemes, morphisms of finite type, finite morphisms, closed immersions, dimension, fibre product
- (4) **Separated And Proper Morphism:** Definition of separated and proper morphisms, valuative criteria, projective morphisms, reduced structure of closed subset, scheme-theoretic image, constructible sets
- (5) **Coherent and Quasi-Coherent Sheaves::** Definition of $\mathcal{O}_X$ -modules, quasi-coherent and coherent sheaves, construction of $\mathcal{O}_X$ -modules, invertible sheaves, vector bundles
- (6) **Divisor:** Weil divisors, Cartier divisors, equivalence for locally factorial schemes, invertible sheaves
- (7) **Projective Morphism:** Criteria and characterization of projective morphisms, ample and very ample sheaves, blowups
- (8) **Differentials:** Derivations, module of relative differential forms, sheaves of differentials and connection to nonsingularity, tangent sheaf, canonical sheaf, geometric genus

1.2. Cohomology.

- (1) **Derived Functor:** Abelian categories, complexes, derived functors, δ -functors
- (2) **Cohomology of Sheaves:** The category of sheaves of $\mathcal{O}_X$ -modules has enough injectives, some basic vanishing theorems
- (3) **Cohomology of Noetherian Affine Scheme:** vanishing of higher cohomology of quasi-coherent sheaves and Serre's criterion for being affine in terms of vanishing of cohomology
- (4) **Cech Cohomology:** Definition, isomorphism with regular cohomology for a noetherian separated scheme
- (5) **Cohomology of Projective Spaces:** : Calculation using Cech cohomology, Serre vanishing, cohomological criterion of ampleness
- (6) **Ext Group and Sheaves:** Definition of basic properties

- (7) **Serre Duality Theorem:** dualizing sheaf and duality for a projective scheme, invertibility of dualizing sheaf for local complete intersection, isomorphism between dualizing sheaf and canonical sheaf for nonsingular projective variety

2. Toric Varieties

- (1) **Definitions:** Convex polyhedral cones, geometry of convex sets, affine toric varieties, fans and toric varieties, toric varieties from polytopes
- (2) **Singularities And Compactness:** Local properties of toric varieties, surfaces, quotient singularities, one-parameter subgroups, limit points, compactness and properness, nonsingular surfaces, resolution of singularities
- (3) **Orbit, Topology And Line Bundles:** Orbits, fundamental groups and Euler characteristics, divisors, line bundles, cohomology of line bundles, canonical class
- (4) **Cohomology of Smooth Toric Varieties:** Stanley-Reisner relations, self-intersection of invariant divisors
- (5) **Reflexive polytopes and Batyrev's duality**