ORAL QUAL SYLLABUS

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(1) **Algebraic Geometry - Schemes**
  (a) Sheaves : Definition, presheaves and associated sheaves, morphisms, stalks, pull-back and push-forward.
  (b) Schemes : Affine Schemes, Proj, structure sheaf, scheme associated to a variety.
  (c) First properties of schemes : Reduced, Integral, Noetherian Schemes, morphisms, closed immersions, dimensions, fibre products.
  (d) Separated and Proper morphisms : Definition of separated and proper morphisms, valuative criteria, projective morphisms, reduced structure of closed subsets, scheme-theoretic image, constructible sets.
  (e) Coherent and Quasi-coherent sheaves : Definition of $\mathcal{O}_X$ modules, quasi-coherent and coherent sheaves, constructions on $\mathcal{O}_X$-modules, invertible sheaves, vector bundles.

(2) **Algebraic Geometry - Cohomology**
  (a) Derived Functors : Abelian Categories, complexes, derived functors, δ-functors.
  (b) Cohomology of Sheaves : The category of sheaves of $\mathcal{O}_X$ modules has enough injectives, some basic vanishing theorems.
  (c) Cohomology of a Noetherian Affine Scheme : vanishing of higher cohomology of quasi-coherent sheaves and Serre's criterion for being affine in terms of vanishing cohomology.
  (d) Cech Cohomology : Definition, isomorphism with sheaf cohomology for a noetherian separated scheme.
  (e) Cohomology of projective space : Calculations using Cech Cohomology, Serre vanishing, cohomological criterion of ampleness.

(3) **Vertex Operator Algebras**
  (a) Definition and Basic Properties of VoAs and their Modules.
    (i) Formal Calculus.
    (ii) Definition of Vertex Algebras and Vertex Operator Algebras and their modules in terms of Jacobi Identity and the Duality Properties.
    (iii) Basic properties, including weak commutativity, weak associativity, commutator formula, associator formula, operator product expansion.
    (iv) Conformal elements.
    (v) General construction theorem for Vertex Operator Algebras.
  (b) Examples of vertex (operator) algebras and their modules
    (i) Vertex (operator) algebras based on Heisenberg Lie algebras
    (ii) Vertex (operator) algebras and based on affine Lie algebras
    (iii) Vertex (operator) algebras based on the Virasoro Algebra
    (iv) Vertex (operator) algebras and modules on even lattices

(4) **Chiral De Rham Complex**

(a) Local definition of Chiral De Rham and N=2 Supersymmetric Structure.
(b) Global Definition of Chiral De Rham, Calabi-Yau Condition
(c) Chiral De Rham of Toric Varieties and Hypersurfaces

References

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