Oral Qualifying Exam Syllabus

Blair Seidler
Committee: Doron Zeilberger (Chair), Bhargav Narayanan,
Michael Saks, Shubhangi Saraf

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1 Combinatorics

- **Enumeration:** Bijections, generating functions, binomial and multinomial coefficients, recurrence relations, inclusion-exclusion, Stirling's formula
- **Hypergraphs:** Sperner, LYM inequality, Erdös-Ko-Rado, Kruskal-Katona, Fisher's Inequality, Ray-Chaudhuri Wilson, Frankl-Wilson, Baranyai, Harper, Sauer-Shelah
- **Posets and Lattices:** Dilworth, Birkhoff representation theorem, distributive lattices, linear extensions of posets, Harris-Kleitman, Fortuin-Kasteleyn-Ginibre (FKG), Ahlswede-Daykin, Möbius inversion
- **Discrepancy:** Beck-Fiala, six standard deviations suffice
- **Ramsey Theory:** Ramsey, infinite Ramsey, upper and lower bounds, probabilistic lower bounds, Van der Waerden, Hales-Jewett, Rado's theorem
- **Linear Programming:** Weak duality theorem, strong duality theorem, fractional coverings and matchings
- **Algebraic Methods:** Combinatorial Nullstellensatz, Schwartz-Zippel Lemma, linear algebraic methods
- **Probabalistic Methods:** Linearity of expectation, union bound, Markov's inequality, Bonferroni inequalities, Chernoff bounds, alteration methods, Chebyshev's inequality, Jensen's inequality

2 Computational Complexity

- **P vs. NP:** Reducibility, Cook-Levin Theorem; NP-completeness of SAT, independent set, 0/1 integer programming, conditions that imply P $\neq$ NP
- **Diagonalization:** Ladner's Theorem, Oracle Turing Machines and the Baker-Gill-Solovay Theorem

- **Space Complexity:** PSPACE completeness of TQBF, NL completeness of PATH, Savitch's Theorem, Immerman- Szelepcsényi Theorem
- **Separation Theorems:** Time and space hierarchy theorems (deterministic and nondeterministic versions)
- **Polynomial Hierarchy:** Σ_i^p;Π_i^p, complete problems, conditions that lead to collapse of PH
- **Circuits:** P$\subseteq$P/poly, Cook-Levin via CKT-SAT, characterization of P/poly as TMs with advice, Karp-Lipton theorem, existence of hard functions, nonuniform hierarchy theorem, definition of AC^i, NC^i
- **Randomization:** RP, BPP, ZPP, error reduction, Sipser-Gács, BPP$\subseteq$P/poly, randomized reductions, definition of BP$\cdot$NP
- **Interactive Proofs:** GNI$\in$AM, NP-completeness of GI implies $\Sigma_2^p = \Pi_2^p$, IP = PSPACE
- **Decision Trees:** Decision tree complexity, certificate complexity, randomized decision tree complexity, sensitivity, block sensitivity, degree, relationships between s(f), bs(f), C(f), D(f), deg(f), and R(f)
- **Circuit Lower Bounds:** Hastad's switching lemma, $ACC0$, Razborov-Smolensky theorem, monotone circuit lower bound for CLIQUE

3 Experimental Mathematics

- **Basics:** Maple coding, dynamic programming
- **Algorithms for Enumerative Combinatorics:** Fixed points, permutations, Gray Codes, set partitions, random walks, graphs, generating functions, posets, Catalan numbers, Prüfer codes, multisets, tableaux

4 Graph Theory

- **Matchings**: König, Hall, Tutte, stable matchings
- **Connectivity**: Menger's Theorem, Max Flow-Min Cut Theorem, Kruskal's Algorithm, Prim's Algorithm, Dijkstra's Algorithm, Matrix-Tree Theorem, Cayley's Formula, structure of 2-connected graphs
- **Coloring:** 5-color theorem, Brooks, Vizing, Thomassen's 5-list-coloring of planar graphs, perfect graphs, Lovász's proof of weak perfect graph theorem, Galvin's proof of Dinitz conjecture
- **Extremal:** Turán's theorem, regularity lemma, counting lemma, Chvatal-Rödl-Szemerédi-Trotter, Erdös-Stone theorem, triangle removal lemma, Roth's theorem