

Oral Qualifying Exam Syllabus

Jason Saied

October 18, 2018

MAJOR TOPIC: FINITE-DIMENSIONAL LIE ALGEBRAS

- **Elementary notions and basic theory**

- Definitions, examples, representations, modules
- Solvable, nilpotent, simple, semisimple Lie algebras and the Killing form
- Engel's Theorem and Lie's Theorem
- Cartan's criteria for semisimplicity and solvability
- Semisimple Lie algebras as direct products of simple Lie algebras
- Weyl's Theorem for complete reducibility of modules for semisimple Lie algebras

- **Semisimple Lie algebras**

- Representations of $\mathfrak{sl}(2, \mathbb{C})$
- Root systems and axiomatics
- Simple roots and the Weyl group
- Classification of root systems

- **Representation theory**

- Universal enveloping algebras
- Poincaré-Birkhoff-Witt Theorem
- Serre's theorem
- Construction of all finite-dimensional modules for semisimple Lie algebras

MINOR TOPIC: VERTEX OPERATOR ALGEBRAS

- **Definitions and properties**

- Formal calculus
- Axiomatic definitions and basic properties of vertex (operator) algebras
- Axiomatic definitions and basic properties of modules for vertex (operator) algebras
- Analytic definitions of vertex operator algebras and modules (as in [2])
- Equivalence of Jacobi identity with commutativity and associativity

- **Examples of vertex operator algebras**

- Huang’s analytic construction theorem for grading-restricted vertex algebras
- Vertex operator algebras associated to the Virasoro algebra (construction using [2], results as in [4], Section 6.1)
- Vertex operator algebras associated to affine Lie algebras (construction using [2], results as in [4], Section 6.2)
- Vertex operator algebras and modules associated to lattices (as in [1])

References

- [1] I. Frenkel, J. Lepowsky, A. Meurman. *Vertex Operator Algebras and the Monster*. Academic Press, Inc., 1989.
- [2] Y.-Z. Huang. *Two constructions of grading-restricted vertex (super)algebras*. J. Pure Appl. Algebra 220 (2016), no. 11, 3628–3649.
- [3] J. E. Humphreys. *Introduction to Lie Algebras and Representation Theory*. Springer, New York, 1972.
- [4] J. Lepowsky, H. Li. *Introduction to Vertex Operator Algebras and their Representations*. Birkhäuser, Boston, 2004.