

Oral Qualifying Exam Syllabus

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1 Combinatorics and Graph Theory

1.1 Combinatorics

- **Enumeration:** Bijections, Generating Functions, Binomial and Multinomial coefficients, Inclusion-Exclusion, Stirling's Formula, Recurrence Relations.
- **Hypergraphs:** Sperner's Theorem, LYM inequality, Erdos-Ko-Rado, Kruskal-Katona, Fisher's Inequality, Baranyai, Ray-Chadhuri-Wilson, Frankl-Wilson and Harper
- **Partially Ordered Sets:** Dilworth's Theorem, linear extensions of posets, Distributive Lattices, Geometric Lattices, Birkhoff Representation Theorem, Dowling-Wilson, Möbius inversion, Weisner.
- **Correlation Inequalities:** Harris- Kleitman, FKG Inequality, Ahlswede-Daykin, Shepp (XYZ)
- **Discrepancy:** Beck-Fiala, Statement of Roth 1/4 theorem on arithmetic progressions
- **Ramsey Theory:** Ramsey, infinite Ramsey, König tree lemma, probabilistic lower bounds, van der Waerden, statement of Szemerédi

1.2 Probabilistic Methods

- **Basics:** Linearity of expectation, union bound, Bonferroni inequalities, Chebychev's inequality, Markov's inequality, Chernoff bounds, alteration methods.
- **Second Moment Method:** Application to threshold function for containing a fixed subgraph
- **Local Lemma:** Symmetric and general versions, applications to hypergraph discrepancy, Ramsey lower bounds
- **Poisson Paradigm:** Janson's inequality and the number of triangles in $G_{n,p}$, Brun's sieve and the number of isolated vertices in $G_{n,p}$

- **Martingales:** Vertex and edge exposures, Azuma's inequality and application to chromatic number, Talagrand's inequality
- **Random Graphs:** $G_{n,p}$ versus $G_{n,M}$, monotone properties, Bollobás-Thomason existence of thresholds
- **Entropy:** Basic properties, Shearer's lemma, Brégman's Theorem

1.3 Graph Theory

- **Matchings:** Hall, König, Tutte, doubly stochastic matrices- Birkhoff von Neumann, matching polytopes
- **Connectivity:** Kruskal and Prim's spanning tree algorithms, Menger's theorem, max-flow min-cut, Cayley's formula and Prüfer Codes, statement of Matrix-Tree
- **Planarity:** Euler's formula, Kuratowski's theorem, Wagner's theorem
- **Coloring:** 5-color theorem, Brooks, Vizing, perfect graphs, statement of Dinitz conjecture/ Galvin's Theorem
- **Extremal:** Turan, regularity lemma, counting lemma, triangle removal lemma

2 Model Theory

- **Basics:** Languages, Structures, Theories, Definability and Interpretability
- **Basic Techniques:** Compactness, Completeness, Henkin Construction, Catagoricity-Vaught's Test
- **Up and Down:** Löenheim-Skolem Theorem, Tarski-Vaught Test, universal axiomatization
- **Back and Forth:** Applications to catagoricity for DLO, Random Graphs- (Zero-One law for Graphs), Ehrenfeucht-Fraïsse Games
- **Types:** Realizing types, Stone topology, statement of Omitting Types Theorem, prime and atomic models