

Oral Qualifying Exam Syllabus

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1 Topoi (MacLane & Moerdijk chapters 1-6)

- General category theory everyone should know
 - Limits and Colimits
 - Exponentials
 - Functors
 - Natural Transformations
 - Adjunctions
 - Whatever I forgot to put here but probably should have
- Grothendieck Topoi
 - Sheaves (of sets) on a Topological Space
 - Sheafification for Topological Spaces
 - Grothendieck Sites
 - Sheaves on a Grothendieck Site
 - Sheafification for Grothendieck Sites ($++$ construction)
- Elementary topoi
 - Definition (Subobject classifiers and the powerset functor)
 - Internal Logic and Heyting Algebras
 - Natural Number Objects
 - The Cumulative Hierarchy in a Topos
 - That Thing With Trees to Build ZF Models in a Topos
- Harder topos stuff you need to do forcing properly
 - Lawvere-Tierney topologies
 - The Dense Below/Double Negation Topology
 - Inclusion functor from $V^{\mathbb{P}}$ to $\text{Sh}(\mathbb{P})$

2 Set Theory (Mostly from Jech)

- Permutation Models and ZFA (see jech chapter 15)
 - consistency of $\text{ZF} + \neg \text{AC}$
 - consistency of $\text{ZF} + \omega_1$ is singular

- Measurable cardinals (from various class notes)
 - definable ultrapowers
 - elementary embeddings
- A bunch of common forcings
 - Cohen forcing
 - Levy collapse
 - Prickry forcing
 - Namba forcing
 - Sacks forcing
 - Shooting a Club
 - Random Reals
- Classic combinatorial set theory
 - Mostowski collapse
 - Skolem hulls
 - L , and proving $L \models \diamond + \text{GCH}$