Syllabus for Oral Exam

Doanh Pham

Major topic: Partial Differential Equations

1. Laplace and Poisson Equations

Mean Value Properties.

Maximum, minimum principles.

Harnack inequality, Liouville Theorem.

Green's representation formula.

Interior estimates for harmonic functions.

Perron's and variational methods for Laplace equations.

Newtonian potential and Holder estimates for Poisson equations.

2. Heat equations

Fundamental solution.

Maximum principle.

Energy method and backward uniqueness.

3. Wave equations

d'Alembert's formula for $n = 1$, Kirchhoff's formula for $n = 3$ and Poisson's formula for $n = 2$.

Energy methods, uniqueness and domain of dependence.

4. Sobolev spaces

Definition and basic properties.

Density of smooth functions.

Trace theorem.

Gagliardo-Nirenberg-Sobolev inequality; Imbedding theorems.

Riesz potential and further imbedding theorems.

Poincare's inequality and difference quotients.

5. Second order elliptic equations

Weak solution.

Lax-Milgram theorem, energy estimates, Fredholm alternatives.

H^2-regularity.

Maximum principle.

Eigenvalues and eigenfunctions of symmetric elliptic operators.

Minor topic: Functional Analysis

6. Banach Spaces

Hahn-Banach theorems.

Banach-Steinhaus theorem, open mapping and closed graph theorems.

Weak and Weak-star topology; Weak and Weak-star convergence.

Double dual; Reflexivity.

7. Hilbert Spaces

Riesz representation theorem in Hilbert spaces.

Lax-Milgram Theorem.

Hilbert sums, orthonomal bases.

8. Compact operators in Hilbert Spaces

Closedness of the space of compact operators; Adjoint.

Fredholm alternative.

Spectral theorems for compact, self-adjoint operators.

References

[1] L. C. Evans, *Partial Differential Equations*, 2nd edition, AMS 2010.

[2] D. Gilbarg and N. Trudinger, *Elliptic Partial Differential Equations of Second Order*, Springer 2001.

[3] H. Brezis, *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, Springer 2011.