

Oral Qualification Exam Syllabus

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Major Topic : Mean Curvature Flow

Basic facts of Mean Curvature Flow

- Definition of solution of mean curvature flow
- Special solutions of mean curvature flow evolving by symmetry
- Diffeomorphism invariance and proof of short time existence and uniqueness of mean curvature flow on closed hypersurfaces
- Evolution of geometric quantities
- Characterization of finite time blow up

Maximum Principle and its Consequences

- Weak maximum principle for scalars and tensors
- Preservation of convex cone in the space of principal curvatures

Convergence to a Single Point

- Gage Hamilton Grayson theorem
- Huisken's theorem

Type I Singularity

- Huisken's monotonicity formula
- Rescaling around type 1 singularity
- Classification of mean convex self shrinker singularity models

Type 2 singularity

- Hamilton's Harnack inequality for mean curvature flow
- Convexity estimate
- Hamilton's blowup procedure and classification of type 2 singularity in mean convex case

Minor Topic : Partial Differential Equations

Laplace Equation / 2nd order Linear Elliptic Equations

- Mean value property of harmonic functions
- Representation formula for solutions of Poisson equation and Green's function
- Weak and strong maximum principles for elliptic equations
- Harnack inequality for elliptic equations

Heat Equation / 2nd order Linear Parabolic Equations

- Mean value property of heat functions
- Representation formula for solutions of heat equation
- Weak and strong maximum principle for parabolic equations
- Harnack inequality for solutions of parabolic equations

Regularity theory of elliptic equations

- L^2 theory
- Moser De Giorgi iteration

References

- [1] B. Andrews, B. Chow, C. Guenther, and M. Langford, *Extrinsic geometric flows*, vol. 206. American mathematical society, 2022.
- [2] K. Ecker, *Regularity theory for mean curvature flow*, vol. 57. Springer Science & Business Media, 2012.
- [3] C. Mantegazza, *Definition and Short Time Existence*, pp. 1–23. Basel: Springer Basel, 2011.

- [4] L. Evans, *Partial Differential Equations*. Graduate studies in mathematics, American Mathematical Society, 2010.
- [5] Q. Han and F. Lin, *Elliptic Partial Differential Equations*. Courant lecture notes in mathematics, Courant Institute of Mathematical Sciences, New York University, 2011.

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