

Oral Qualifying Exam Syllabus

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1. Automorphic Forms

(a) The Modular Group

- i. $SL(2, \mathbb{Z})$ and Congruence Subgroups
- ii. Fundamental Domains for $SL(2, \mathbb{Z})$ and congruence subgroups
- iii. Cusps and Elliptic Points
- iv. The invariant measure of $\mathcal{H}$ under $SL(2, \mathbb{Z})$

(b) Modular Forms

- i. Modular and cusp forms
- ii. Fourier expansions
- iii. The dimensions of $M_k(\Gamma(1))$ and $S_k(\Gamma(1))$
- iv. Eisenstein Series, the Dedekind η function, Δ , and the Jacobi triple product formula
- v. The Petersson inner product on $S_k(\Gamma(1))$
- vi. The L-functions for modular form and functional equations

(c) Hecke Operators

- i. The slash and Hecke operators on holomorphic functions
- ii. Coset representatives for $SL(2, \mathbb{Z}) \backslash M_n(\mathbb{Z})$
- iii. Commutativity and self-adjointness of the Hecke operators
- iv. Hecke eigenforms and Fourier coefficients
- v. Euler products for Hecke eigenforms

(d) The Rankin-Selberg Method

- i. The nonholomorphic Eisenstein series
- ii. Analytic continuations and Euler products for the convolution of two modular forms

(e) Twisting and Converse Theorem

2. Analytic Number Theory

(a) Poisson Summation

(b) The Mellin transform and the Γ function

(c) The Phragmen-Lindelöf principle and convexity bounds

- i. Sub-convexity for $\zeta(s)$

- (d) The Riemann ζ function and Dirichlet L-functions
 - i. Euler Products
 - ii. θ functions
 - iii. Analytic continuation and functional equations
- (e) Dirichlet's Theorem
- (f) The Prime Number Theorem
- (g) The Explicit Formula for $\psi(x)$
- (h) Brun's Theorem via Selberg upper bound sieve

References:

- Automorphic Forms and Representations by Bump
- Multiplicative Number Theory by Davenport
- Opera de Cribro by Friedlander and Iwaniec
- Automorphic Forms and L-Functions for the Group $GL(n, R)$ by Goldfeld
- Analytic Number Theory by Iwaniec and Kowalski