

Oral Qualifying Exam Syllabus

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Major: Optimization theory

Convex Optimization

1. Unconstrained optimization [Ahm, §3]
 - Applications: least squares, detecting feasibility
 - Descent directions and first order optimality conditions
 - Second order optimality conditions
2. Convex sets (and functions) [Ahm, §4]
 - Definition and mid-point convexity and examples
 - Sublevel sets, quasiconvexity and epigraphs
 - Convex hull and Caratheodory's theorem
3. Separating hyperplane and LP duality [Ahm, §5]
 - Separating hyperplane theorems
 - Farkas Lemma
4. An application of LP strong duality to combinatorial optimization [Ahm, §6]
 - Bipartite matching and vertex covers
 - König's theorem
 - Totally unimodular matrices and integral polytopes
5. Convex functions [Ahm, §7]
 - Convex, concave, strictly convex, and strongly convex functions
 - First and second order characterizations of convex functions
 - Optimality conditions for convex problems
6. Convexity-preserving operations [§2 Par; Ahm, §8]
7. Convex envelopes, cardinality constrained optimization and LASSO [Ahm, §8]
8. LP, QP, QCQP, SOCP, SDP [Ahm, §9]
9. Applications of SDP [Ahm, §10, 11]
 - Stability and stabilizability of linear systems
 - Eigenvalue and matrix norm minimization problems
 - Independent/stable sets, Shannon capacity of a graph

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Algorithmic Algebraic Geometry

1. Solving polynomial equations
 - Solving Polynomial Systems by Elimination [CLO05, §2.1]
 - Solving Equations via Eigenvalues and Eigenvectors [CLO05, §2.5]
2. Resultants
 - The resultant of two polynomials and multipolynomial resultant [CLO05, §3.1, 3.2]
 - Resultants, discriminants, applications [Par, §6]
 - Properties of resultants and computing them [CLO05, §3.3, 3.4]
 - Solving Equations via Resultants [CLO05, §3.5]
3. Polytopes, Resultants, and Equations
 - Geometry of polytopes [CLO05, §7.1]
 - Minkowski Sums and Mixed Volumes [CLO05, §7.4]
 - Bernstein's Theorem [CLO05, §7.5]
 - Computing Resultants and Solving Equations [CLO05, §7.6]
4. Binomial Equations, Newton Polytopes, The Bézout and BKK Bounds, Application to Nash Equilibria [Par, §9]
5. Nonnegativity, SOS, Semidefinite Programming [Par, §10]

Minor: Probability Theory

1. Measure theory
 - Probability Spaces [Dur19, §1.1]
 - Distributions [Dur19, §1.2]
 - Random Variables [Dur19, §1.3]
 - Integration [Dur19, §1.4]
 - Expected value [Dur19, §1.6]
2. Laws of Large Numbers
 - Independence [Dur19, §2.1]
 - Weak Laws of Large Numbers [Dur19, §2.2]
 - Borel-Cantelli Lemmas [Dur19, §2.3]
 - Strong Law of Large Numbers [Dur19, §2.4]
3. Central Limit Theorems
 - Weak Convergence [Dur19, §3.2]
 - Characteristic Functions [Dur19, §3.3]
 - Central Limit Theorems [Dur19, §3.4]
4. Concentration inequalities
 - Chernoff bound [Wai19, §2.1.1]
 - Sub-Gaussian variables and Hoeffding bounds [Wai19, §2.1.2]
 - Sub-exponential variables and Bernstein bounds [Wai19, §2.1.3]

References

- [CLO05] D. A. Cox, J. B. Little, and D. O'Shea. *Using Algebraic Geometry*. Second. Vol. 185. Graduate Texts in Mathematics. Springer, 2005. URL: <https://doi.org/10.1007/b138611>.
- [Dur19] R. Durrett. *Probability: theory and examples*. Vol. 49. Cambridge university press, 2019. URL: https://services.math.duke.edu/~rtd/PTE/PTE5_011119.pdf.
- [Wai19] M. J. Wainwright. *High-Dimensional Statistics: A Non-Asymptotic Viewpoint*. Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press, 2019.
- [Ahm] A. A. Ahmadi. *Lecture notes on Convex and Conic Optimization*. URL: <https://aaa.princeton.edu/orf523>.
- [Par] P. Parrilo. *Lecture notes on Algebraic Techniques And Semidefinite Optimization*. URL: <https://ocw.mit.edu/courses/6-972-algebraic-techniques-and-semidefinite-optimization-spring-2006/>.