Syllabus for the Oral Qualifying Exam

Anthony Matos

Major Topic: Elliptic Partial Differential Equations

- Laplace's equation, Poisson's equation, basic properties of harmonic functions

1. Mean value property and applications: the maximum principle, Harnack's inequality, regularity of harmonic functions
2. The fundamental solution, Green's identities, Green's function, Poisson integral formula, Harnack's convergence theorem, Bocher theorem on removable singularities
3. Bernstein's method for gradient estimates and applications: Harnack inequality
4. Potential theory and C^{2,\alpha} estimates for solutions of the Poisson equation
5. Calderon-Zygmund inequality

- Classical solutions of nondivergence form elliptic equations

1. The weak maximum principle, Hopf boundary point lemma, the strong maximum principle, application: apriori estimates
2. The interior Schauder estimate
3. Boundary Schauder estimate
4. The Dirichlet problem, method of continuity

- Sobolev Spaces

1. Definition and basic properties
2. Approximations by smooth functions, Chain rule
3. Gagliardo-Nirenberg inequality, Morrey's inequality, Poincare inequality
4. Sobolev embedding theorem via extensions
5. Rellich-Kondrachev compact embedding theorem
6. Interpolation inequality.

- Weak solutions to elliptic equations of divergence form

1. Lax-Milgram and Fredholm Alternative, application: existence of weak solutions

2. Local boundedness of weak solutions: De-Giorgi and Moser's iteration proof
3. Oscillation estimate and Hölder continuity
4. John-Nirenberg inequality, BMO, Moser's Harnack inequality

Minor Topic: Riemannian Geometry

- Riemannian metrics and the Levi-Civita connection

1. Local representation of metrics, transformation behavior, Christoffel symbols
2. Covariant derivative and parallel transport
3. Uniqueness of Levi-Civita connection

- Curvature tensors

1. Riemann curvature tensor, sectional curvature, Ricci and scalar curvature
2. Bianchi identities, Schur's theorem

- Geodesics

1. Local existence, homogeneity
2. The exponential map and normal coordinates
3. Gauss's lemma, local minimizing property of geodesics
4. Normal and uniformly normal neighborhoods
5. Complete manifolds and Hopf-Rinow theorem

- Jacobi fields

1. Definitions and basic properties
2. First and second variations of energy
3. Bonnet-Myers and Synge theorem