

Oral Qualifying Exam Syllabus

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(Chair)

1 Major Topic: Mean Curvature Flow

The Basics

- First variation of area functional
- Definitions of the flow: Standard definition, invariance under tangential perturbation, definition for graphs
- Special Solutions: shrinkers, expanders, translating solutions
- Proof of short time existence

The Maximum Principle and its Consequences

- The Maximum Principle (for Mean Curvature Flow)
- The Comparison Principle
- The evolution of geometric quantities under mean curvature flow
- Consequences of the evolution equations
- Preservation of: Convexity, mean convexity, k -convexity, etc

Huisken's Monotonicity Formula and its Consequences, Type 1 Singularities

- Huisken's Monotonicity Formula
- Rescaling around type 1 singularities
- Analysis of Singularities
- Singularity Models for Mean Convex Hypersurfaces

Type 2 Singularities

- Hamilton's Blow-up

- The Mean Convex Case
- Hamilton's Harnack Estimates for Mean Curvature Flow

Regularity Theory at the First Singular Time

- Lower Bound on Area Ratio / Clearing Out Lemma
- White's Gap Theorem
- Brakke's Regularity Theorem

2 Minor Topic: Partial Differential Equations

- Laplace's Equation
 - Representation formula for the solution of the dirichlet problem on the ball
 - Mean Value Property for subharmonic functions
 - Other Properties of Harmonic Functions: Strong Maximum Principle, regularity, Harnack's inequality, Green's function
- The Heat Equation
 - Maximum Principles
 - Fundamental Solution of the Heat Equation
 - Other properties of solutions: Strong maximum principle, uniqueness of solution
- Second-Order Elliptic Equations
 - Hopf's Maximum Principle
 - Alexandrov and Bakelman's Maximum Principle
 - Maximum Principles for Nonlinear PDE

3 References

- [E] Klause Ecker, Regularity Theory for Mean Curvature Flow
 [J] Jurgen Jost, Partial Differential Equations (Second Edition) Ch 1, 2, 4
 [M] Carlo Mantegazza, Lecture Notes on Mean Curvature Flow