

Syllabus of Oral Qualifying Exam

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Hamilton's Ricci Flow

- **Short Time Existence**
 - Evolution of geometric quantities under the flow
 - Symbol of differential operators, parabolic linear and semilinear equations
 - DeTurck's trick
- **Maximum Principle and Applications**
 - Scalar and tensor maximum principles for parabolic equations
 - BBS estimates for derivatives of curvature
 - Doubling time estimates for Ricci flow
- **Normalized Ricci flow on Surfaces**
 - Uniqueness of Ricci solitons
 - Entropy estimates
 - Derivative and maximum estimates for scalar curvature
 - Differential Harnack estimates
- **Ricci Flow on 3-Manifolds**
 - Pinching and Hamilton-Ivey estimates for eigenvalues of curvature
 - Estimates for trace-free Ricci curvature
 - Hamilton's gradient estimate
 - Convergence of normalized flow

Second Order Linear Elliptic and Parabolic PDEs

- **Poisson and Non-Homogeneous Heat Equations**

- Mean value property for Laplace and Heat equation
- Interior derivative estimates for Laplace and Heat equation
- Harnack inequality for harmonic functions
- Newtonian potential
- Poisson representation formula
- Explicit solution of initial value non-homogeneous heat equation

- **Maximum principle**

- Classical weak maximum principle for elliptic and parabolic equations
- Hopf's boundary point lemma and strong maximum principle
- Maximum principle for weak solutions of elliptic equations
- Bernstein derivative estimates for heat equation

- **Regularity**

- Global Sobolev regularity for elliptic and parabolic equation
- Global Schauder estimate for elliptic equations
- Interior Schauder estimate for parabolic equations
- Global Holder regularity for elliptic equations

- **Existence Methods**

- Energy method for elliptic and parabolic equations
- Galerkin method for parabolic equations
- Continuity method
- Perron's method