

# Oral Qualifying Exam Syllabus

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## 1 Geometry

### 1.1 Convex Geometry

1. Basic Definitions of Convex Geometric Objects [Gru07]
2. Carathéodory, Helly, and Radon's theorems [Gru07]
3. Hyperplane separation theorem [Gru07]
4. Koebe-Andreev-Thurston Theorem [Gru07]

### 1.2 Differential Geometry

1. Tangent Spaces and Derivatives [Lee12]
2. Submanifolds of Euclidean Space and Embeddings [Lee12]
3. Vector Fields and Flows [Lee12]
4. Lie Groups [Lee12]
5. Riemannian Metrics [Lee19]

### 1.3 Combinatorial Geometry

1. Incidence Geometry: Erdős-Szekeres Theorem, Szemerédi-Trotter Theorem [PA95]
2. Geometry of Numbers: Blichfeldt's Lemma, Steinitz's Theorem [PA95]
3. Lattices: Minkowski's Theorem for General Lattices, Two Squares Theorem [PA95]

## **1.4 Computational Geometry**

1. Linear Programming [Ber+97]
2. Computing Voronoi Diagrams [Ber+97]
3. Computing Convex Hulls [Ber+97]
4. Delaunay Triangulation [Ber+97]
5. Well-separated Pairs [Har11]
6. Quadrees [Har11]
7. Bourgain's Theorem [Bou85]

## **2 Graph Theory**

### **2.1 Basic Graph Theory**

1. Bondy-Chvatal Theorem [Bol79]
2. Ramsey Theory: Ramsey's Theorem, General Ramsey's Theorem [Bol79]
3. Max-flow Min-cut theorem [Bol79]
4. Konig's Theorem [Bol79]
5. Vertex Coloring, Edge Coloring [Bol79]
6. Dilworth's Theorem [Bol79]
7. Perfect Graph Theorem [Bol79]
8. Menger Theorem [Bol79]
9. Tutte's Embedding Theorem [Ste]
10. Tutte-Polynomial, Chromatic-Polynomial [Bol79]
11. Szemerdi's Regularity Lemma [Adr10]

### **2.2 Extremal Graph Theory**

1. Turan's Theorem, Mantel's Theorem [Bol79]
2. Erdos-Stone Theorem [Bol79]

### 3 References

#### References

- [Adr10] U.S.R Murty Adrian Bondy. *Graph Theory*. Graduate Texts in Mathematics. Springer London, 2010. ISBN: 978-1-84996-690-0.
- [Ber+97] M. de Berg et al. *Computational Geometry*. Springer-Verlag Berlin Heidelberg, 1997. ISBN: 3-540-65620-0.
- [Bol79] Béla Bollobás. *Graph Theory*. Graduate Texts in Mathematics. Springer-Verlag New York, 1979. ISBN: 0-387-90399-2.
- [Bou85] J. Bourgain. “On lipschitz embedding of finite metric spaces in Hilbert space”. In: *Israel Journal of Mathematics* (1985), pp. 46–52.
- [Gru07] Peter M. Gruber. *Convex and Discrete Geometry*. A Series of Comprehensive Studies in Mathematics. Springer-Verlag Berlin Heidelberg, 2007. ISBN: 978-3-540-71132-2.
- [Har11] Sarel Har-Peled. *Geometric Approximation Algorithms*. USA: American Mathematical Society, 2011. ISBN: 0821849115.
- [Lee12] John M. Lee. *Introduction to Smooth Manifolds*. Graduate Texts in Mathematics. Springer New York, NY, 2012. ISBN: 978-1-4419-9982-5.
- [Lee19] John M. Lee. *Introduction to Riemannian Manifolds*. Graduate Texts in Mathematics. Springer Cham, 2019. ISBN: 978-3-319-91755-9.
- [PA95] János Pach and Pankaj K. Agarwal. *Combinatorial Geometry*. Series in Discrete Mathematics and Optimization. John Wiley & Sons, Incorporated, 1995. ISBN: 0-471-58890-3.
- [Ste] Dylan Thurston Steveb J. Gortler Craig Gotsman. “Discrete One-Forms and Applications to 3D Mesh Parameterization”.