

Oral Qualifying Exam Syllabus

Michail Lolis

Analytic Number Theory

1. Dirichlet characters and Gauss sums.
2. Riemann zeta function and Dirichlet L-functions, their analytic continuation, Euler product, explicit formula and convexity bound.
3. Poisson summation and Mellin transform.
4. Dirichlet's Theorem on primes in arithmetic progression.
5. The Prime Number Theorem for primes in arithmetic progression.
6. The Large Sieve and Bombieri's Theorem.

Modular Forms

1. Congruence subgroups of $SL_2(\mathbb{Z})$ and their fundamental domains.
2. Modular forms and cusp forms of integer weight with respect to some congruence subgroup Γ .
3. Dimension formulas for spaces of modular forms.
4. Eisenstein and Poincaré Series, the Jacobi theta function, the modular function j and the Δ function.
5. Hecke operators. Commutativity and self-adjointness with respect to Petersson inner product.
6. L-functions of modular forms and their functional equations.

Algebraic Number Theory

1. Number Fields, their ring of integers and integral bases.
2. The trace, norm and discriminant.
3. Prime decomposition in Number rings, fractional ideals, ramification and splitting. Norm of elements and ideals.

4. The Class group and the class number, Dirichlet's unit Theorem and Minkowski's bound.
5. Dedekind zeta function and the class number formula.

References

- [Dav00] Harold Davenport, *Multiplicative number theory*, 3 ed., Graduate Texts in Mathematics, vol. 74, Springer, New York, 2000, Revised by H. L. Montgomery.
- [DS05] Fred Diamond and Jerry Shurman, *A first course in modular forms*, Graduate Texts in Mathematics, vol. 228, Springer, New York, 2005.
- [IK04] Henryk Iwaniec and Emmanuel Kowalski, *Analytic number theory*, American Mathematical Society Colloquium Publications, vol. 53, American Mathematical Society, Providence, RI, 2004.
- [Iwa97] Henryk Iwaniec, *Topics in classical automorphic forms*, Graduate Studies in Mathematics, vol. 17, American Mathematical Society, Providence, RI, 1997.
- [Mar77] Daniel A. Marcus, *Number fields*, Springer-Verlag, New York, 1977.