

Oral Qualifying Exam Syllabus

Yiyang Liu

September 2025

Symplectic Geometry

Basic Constructions

1. Symplectic Vector Spaces
2. Symplectic Structure on Manifold
3. Cotangent Bundle
4. Lagrangian Submanifold

Local Theory

1. Moser Theorems
2. Darboux-Moser-Weinstein Theory
3. Weinstein Tubular Neighborhood Theorem

Almost Complex Structure

1. Existence of compatible Almost Complex Structure
2. Complex manifolds
3. Kähler forms
4. Basics of Dolbeault Theory

Hamiltonian Dynamics and Moment Maps

1. Hamiltonian Vector Fields
2. Symplectic and Hamiltonian Actions
3. Moment maps
4. Marsden-Weinstein Reduction
5. Convexity of Moment Map

J-holomorphic Curves

1. Mean Value Inequality
2. Isoperimetric Inequality
3. Exponential Decay on Long Cylinders

Algebraic Topology

Homology

1. Singular Homology
2. Homotopy Invariance
3. Long Exact Sequence
4. Excision
5. Degree Theory
6. Cellular Homology
7. Mayer Vietoris Sequence

Cohomology

1. Universal Coefficient Theorem
2. Cohomology Ring
3. Kunneth Formula
4. Poincare Duality

References

- [1] Allen Hatcher, *Algebraic Topology*, Cambridge University Press.
- [2] Ana Cannas da Silva, *Lectures on Symplectic Geometry*, Springer, Lecture Notes in Mathematics 1764, 2008.
- [3] Dusa McDuff, Dietmar Salamon, *J-holomorphic Curves in Symplectic Topology*, AMS Colloquium Publications Volume 52.