Oral Exam Syllabus

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Major Topic

Algebraic Topology

[Hatcher 2002]

- Fundamental Group
    1. Homotopy and fundamental groups
    2. Seifert-Van Kampen's theorem
    3. Covering spaces, deck transformations and group actions
- Homology
    1. Simplicial homology and singular homology
    2. Exact sequences and excision
    3. Cellular homology
    4. Mayer-Vietoris sequences
    5. Simplicial approximation

Minor Topics

Riemannian Geometry

[Do Carmo and Flaherty Francis 1992]

- Riemannian metrics
- Connection
    1. Affine connection, parallel transport

    2. Levi-Civita connection
- Geodesics
    1. Exponential map
    2. Gauss lemma
    3. minimizing properties of geodesics
- Curvature
    1. Sectional, Ricci and scalar curvature
    2. Second fundamental form and Gauss equation
- Hyperbolic geometry
    1. Upper-half plane model
    2. Hyperbolic geodesics and isometries
    3. Disk model and the Ptolemy identity
    4. Hyperboloid model

Riemann Surface

[Forster 2012, Donaldson 2011]

- Elementary properties of holomorphic mappings
- Calculus on surfaces
    1. Differential forms
    2. de Rham cohomology
        (a) Definitions and examples
        (b) Cohomology with compact support
    3. Calculus on Riemann surfaces
- Applications of Euler characteristic
    1. The Riemann-Hurwitz formula
    2. The degree-genus formula
- Riemann-Roch theorem
- Weyl's lemma
- The uniformisation theorem

References

Do Carmo, Manfredo Perdigao and J Flaherty Francis (1992). *Riemannian geometry*. Vol. 2. Springer.

Donaldson, Simon (2011). *Riemann surfaces*. Oxford University Press.

Forster, Otto (2012). *Lectures on Riemann surfaces*. Vol. 81. Springer Science & Business Media.

Hatcher, Allen (2002). *Algebraic topology*. Cambridge: Cambridge University Press, pp. xii+544. ISBN: 0-521-79160-X; 0-521-79540-0.