

Oral Qualifying Exam Syllabus

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Main Topic: Representation Theory of Finite Groups

1. Group Algebras
 - (1) Schur's Lemma
 - (2) Wedderburn's Theorem
2. Complex Representations and Characters
 - (1) Schur Orthogonality Relations
 - (2) Integrality of Characters and Central Characters
 - (3) Tensor Products
 - (4) Frobenius-Schur Indicator
3. Induction and Restriction
 - (1) Frobenius Reciprocity
 - (2) Clifford's Theorem
 - (3) Clifford Correspondence and Gallagher's Theorem
4. Representations over Subfields of $\mathbb{C}$
 - (1) Absolutely Irreducible Representations and Splitting Fields
 - (2) Schur Index
5. Projective Representation
 - (1) Central Extensions
 - (2) Schur Multiplier
 - (3) Character Triples

Minor Topic: Lie Algebras

1. Basic Concepts of Lie Algebras
 - (1) Ideals, Homomorphisms, Representations, and Derivations
 - (2) Solvable and Nilpotent Lie Algebras
 - (3) Engel's Theorem
2. Semisimple Lie Algebras
 - (1) Lie's Theorem and Cartan's Criterion
 - (2) Killing Form
 - (3) Completely Reducible Representations and Weil's Theorem
 - (4) Root Systems
 - (5) Simple Roots and Weyl Group
 - (6) Classification of (Semi)simple Lie Algebras
3. Representation Theory
 - (1) Universal Enveloping Algebras
 - (2) Poincaré-Birkhoff-Witt Theorem
 - (3) Serre's Theorem

References

- [1] I. M. Isaacs, *Character theory of finite groups*, Academic Press, New York, 1976. MR0460423
- [2] J. E. Humphreys, *Introduction to Lie algebras and representation theory*, Graduate Texts in Mathematics, 9, Springer-Verlag, New York, 1978. MR0499562