Syllabus for Oral Qualifying Exam

Heejin Lee

1. Partial Differential Equations

(a) Laplace/Poisson Equation

(1) Fundamental Solution

(2) Mean Value Formulas and Converse to Mean-Value Property

(3) Properties of Harmonic Functions: Maximum principle, Smoothness, Local estimates, Liouville's theorem, Analyticity, Harnack's inequality, Removable Singularity, Schwarz Reflection Principle

(4) Green's functions for a Ball and for a Half-Space

(5) The Classical Dirichlet Problem by Perron's method

(6) The Energy Estimates

(b) Heat Equation

(1) Fundamental Solutions

(2) Maximum Principle on Bounded $\overline{U}_T$ and $\mathbb{R}^n \times [0, T]$, the Uniqueness by Maximum Principle

(3) Uniqueness and Backward Uniqueness by Energy Method

(c) Wave Equation

(1) Solutions by Spherical Means for Dimension $n = 1, 2$ and 3: d'Alembert's Formula for $n = 1$, Kirchhoff's Formula for $n = 3$, and Poisson's Formula for $n = 2$.

(2) Uniqueness and Domain of Dependence by Energy Methods

(d) Sobolev Space

(1) Definition of Sobolev Space

(2) Approximation by Smooth Functions

(3) Extensions and Traces

(4) Sobolev Inequalities: Gagliardo-Nirenberg-Sobolev inequality, Morrey's Inequality

(5) Compact Embedding Theorem: Rellich-Kondrachov Theorem

(6) Poincare's Inequality, Difference Quotients

(7) The Space H^{-1}

(8) Fourier Transform Method for $H^k(\mathbb{R}^n)$

(e) Second Order Elliptic Equations

(1) Definition of Weak Solutions

(2) Existence by Lax-Milgram Theorem and Energy Estimates by Fredholm Alternative

(3) Interior Regularity and Boundary Regularity

(4) Weak and Strong Maximum Principles

(5) Harnack's Inequality

2. Functional Analysis

(a) Banach Spaces

(1) Metric Spaces, Normed Linear Vector Spaces, Banach Spaces and Hahn-Banach Theorem

(2) The Baire Category Theorem, the Uniform Boundedness Principle, the Open Mapping Principle and the Closed Graph Theorem, Banach-Alaoglu Theorem

(3) Duality, Reflexivity, Weak Topology and Weak* Topology

(b) Hilbert Spaces

(1) Definition of Hilbert Space and the Dual Space of Hilbert Space

(2) Projection Lemma, Riesz Representation Theorem and Lax-Milgram Theorem

(3) Orthogonal Sets, Basis, the Bessel's Inequality and the Parseval's Theorem

(c) Compact Operators

(1) Definition, Elementary Properties and Adjoint

(2) The Riesz-Fredholm Theory

(3) The Spectrum of a Compact Operator

References

[1] Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.

[2] Haim Brezis, *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, Springer, 2010.