

# Rutgers Oral Qualifying Exam Syllabus

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## 1 Major: Complex geometry

Reference: Griffiths and Harris, *Principles of Algebraic Geometry*

### 1.1 Complex Manifolds

- De Rham and Dolbeault Cohomology
- De Rham Theorem and Dolbeault Theorem
- Submanifolds and Subvarieties

### 1.2 Kähler Manifolds

- Hodge Decomposition for Kähler Manifolds
- Kodaira-Serre Duality
- $\bar{\partial}\partial$ -Lemma on Kähler Manifolds
- Lefschetz decomposition
- Distributions and Currents

### 1.3 Kodaira Embedding Theorem

- Curvature of Line Bundles
- Chern Class
- Adjunction Formulas
- Chern Classes of Line Bundles
- Kodaira Vanishing Theorem and Lefschetz Theorem
- Blow Up

## **2 Minor: Representation Theory of Semisimple Lie Algebras**

### **2.1 Lie Algebras**

- Basic Definitions
- Semisimple Lie Algebras and Root systems

### **2.2 Universal Enveloping Algebra**

- Universal Enveloping Algebra Center
- Poincare-Birkhoff-Witt Theorem

### **2.3 Representation Theory**

- Highest Weight Modules and Verma Modules
- Weyl's complete reducibility theorem
- Classification of finite-dimensional irreducible modules
- Weyl's character formula