Oral Exam Syllabus

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1 Major : Minimal Surface Theory and Riemannian Geometry

- Basic Riemannian Geometry
  - Curvature: Sectional, Ricci, Scalar Curvature, Mean Curvature and Gauss Curvature
  - Jacobi Field, Myers Theorem and Synge Theorem and Morse Index Theorem
  - Comparison Thoerem : Rauch Comparison Theorem, Volume Comparison Thoerem, Topogonov Comparison thoerem
  - Morse Theory : Morse Lemma, Handlebody Decomposition Theorem, Morse Inequality
  - Bochner's Formula
- Variation Formula and Consequences
  - Definition and Minimal Surface Equation in $\mathbb{R}^n$
  - First Variation Formula and Monotonicity Formula
  - The Theorem of Bernstein
  - The Strong Maximum Principle
  - Second Variation Formula, Morse Index, and Stability
- Curvature Estimates and Compactness Theorem
  - Basic Compactness Theorem
  - 4π Estimate, Small Energy Curvature Estimate
  - Global and Local Estimate
  - Total Curvature Estimate
  - Minimal Cone

- Minimal Surfaces in Three-Manifolds
  - Hersch's and Yang and Yau's Theorems
  - Lower bound for the first eigenvalue
  - Compactness Theorems with A Priori Bounds
  - Positive Mass Theorem
  - Extinction of Ricci Flow
- Yau's conjecture : The abundance of minimal hypersurfaces on compact manifold
  - Existence of Infinitely many minimal hypersurfaces in positive ricci curvature (with Frankel Property)
  - Existence of Infinitely many minimal hypersufraces in closed manifold with general metric
  - Sublinear Growth of k-width of mod 2 cycle (Guth's Argument)

2 Minor : Partial Differential Equations

- Laplace's equation
  - Fundamental Solution of Laplace's equation
  - Mean-value formula
  - Properties of Harmonic function
  - Green's function
  - Harnack Inequality
- Heat Equation
  - Fundamental Solution
  - Strong Maximum Principle
  - Uniqueness

- Existence and Maximum principle of linear elliptic PDEs
  - Existence of Weak Solutions
  - Maximum principles for weak solutions of elliptic PDEs
  - Eigenvalues and Eigenfunctions
- Calculus of Variations
  - First and Second Variation, Euler-Lagrange Equations
  - Existence of Minimizers
  - Mountain Pass Theorem

References

[1] T. H. Colding and C. De Lellis, The min-max construction of minimal surfaces, in Surveys in differential geometry, Vol. VIII (Boston, MA, 2002), Surv. Differ. Geom., VIII, Int. Press, Somerville, MA, 2003, pp. 75–107

[2] T. H. Colding, W. P. Minicozzi, A course in minimal surfaces. Graduate studies in mathematics vol. 121. American Mathematical Society, 2011.

[3] M. do Carmo, Riemannian geometry, Mathematics: Theory & Applications. Birkhäuser Boston, Inc., Boston, MA, 1992.

[4] L. C. Evans, Partial Differential Equations, 2nd edition, AMS 2010.

[5] L. Guth. Minimax problems related to cup powers and steenrod squares. Geom. Funct. Anal., 18:1917–1987, 2009

[6] J. Jost, Riemannian Geometry and Geometric Analysis, seventh edition , Springer, 2011.

[7] F. C. Marques and A. Neves. Existence of infinitely many minimal hypersurfaces in positive Ricci curvature. Invent. Math., 209(2):577–616, 2017.

[8] J. Milnor. Morse theory, volume 51 of Annals of Mathematics Studies. Princeton University press, Princeton, New Jersey, 1963.

[9] A. Song, Existence of infinitely many minimal hypersurfaces in closed manifolds, arXiv:1806.08816 [math.DG] (2018)

[10] B. White, Lectures on Minimal Surface Theory, arXiv:1308.3325.