

Oral Qualifying Exam Syllabus

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1. Elliptic Curves

- The group law on elliptic curves.
- Isogenies of elliptic curves and j -invariant.
- Elliptic curves over $\mathbb{C}$, elliptic functions.
- Elliptic curves over finite fields. The Hasse bound.
- Mordell-Weil Theorem over $\mathbb{Q}$.
- Reduction of rational elliptic curves mod p .

2. Modular Forms

- Modular forms and cusp forms for the full modular group. Dimension of spaces. Examples (Eisenstein Series, Δ function, j -function.)
- Fourier expansion of modular forms. Trivial bound on coefficients.
- L -functions for modular forms and functional equations.
- Hecke theory for the full modular group. Hecke operators. Hecke eigenforms.

3. Analytic Number Theory

- Riemann zeta function and Dirichlet L -functions. Euler product. Analytic continuation. Functional equations.
- Primes in arithmetic progressions
- Zero-free region for $\zeta(s)$. Counting zeros. Prime Number Theorem. Explicit formula for $\psi(x)$.
- Zero-free region for Dirichlet L -function. Siegel's theorem on real zeros of $L(s, \chi)$, where χ is real.

References

- [1] M. R. Murty. *Problems in Analytic Number Theory*.
- [2] M. R. Murty, M. Dewar and H. Graves. *Problems in the Theory of Modular Forms*.
- [3] J. H. Silverman. *The Arithmetic of Elliptic Curves*.
- [4] H. Iwaniec. *Topics in Classical Automorphic Forms*.
- [5] H. Davenport. *Multiplicative Number Theory*.