

Oral Qualifying Exam Syllabus

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1. Symplectic Geometry

1)**Symplectic Manifolds**: Symplectic Vector Spaces, Symplectic Structure on Manifold, Tautological Symplectic form on Cotangent Bundle, Symplectic Volume, Lagrangian Submanifold, Symplectic and Hamiltonian Isotopies

2)**Local Theory of Symplectic Manifolds**: Moser's Trick, Weinstein Lagrangian Neighborhood, Darboux Coordinates, Isotopic and Strongly Isotopic Symplectic Structures

3)**Almost Complex Structure**: Existence of compatible Almost Complex Structure, Complex manifolds, Kähler forms, basics of Dolbeault Theory.

4)**Hamiltonian Dynamics and Moment Maps Hamiltonian Vector fields**, Symplectic and Hamiltonian actions, Moment maps, Orbit spaces, Marsden-Weinstein Reduction, Convexity of Moment map, Symplectic Structure on the Space of Connections, Action of Gauge group (Example of Circle Bundle), Existence and Uniqueness of Moment Maps

References:

- 1) Lectures on Symplectic Geometry, Ana Cannas da Silva
- 2) Introduction to Symplectic Topology, McDuff and Salamon

2. Riemannian Geometry

- (1) Riemannian metric
- (2) Levi-Civita connection
- (3) Curvature tensor, sectional curvature, Ricci curvature, scalar curvature
- (4) Geodesics and exponential map

(5) Jacobi fields, conjugate points

(6) First variation and second variation formulas and consequences,

(7) Bonnet-Myers Theorem, Hopf-Rinow Theorem, Cartan-Hadamard Theorem, Synge Theorem,

Reference:

1) Riemannian Geometry by Manfredo P do Carmo

2) Introduction to Smooth Manifolds by John M Lee.